Oscillations of synchronization in inhibitory coupled Hodgkin-Huxley neurons network

Andrey Andreev
Neuroscience and Cognivite Technology Laboratory,
Center for Technologies in Robotics
and Mechatronics Components
Innopolis University
Innopolis, Russia
andreevandrei1993@gmail.com

***Abstract*—We investigate the dynamics of complex network of Hodgkin-Huxley neurons. It consists of 3 sub-networks. The first small network receives external signal which is transferred into a spike sequence. Then it is transmitted to two small-world networks interacting via an inhibitory coupling and working together to process the signal. We observe that the synchronization index (SI) in both networks periodically changes in time: the time intervals with the high SI alternate with the time intervals where SI is low. We calculate correlation between them and find that when adjusting the strength of the inhibitory coupling one can observe SI in these networks changes either in phase or out of phase.**

Keywords—Neural network, synchronization, small-world network, Hodgkin-Huxley model

I. INTRODUCTION

The dynamics of complex networks has attracted much attention in recent years [1-3]. Especially, the networks of spiking neurons or neuron-like elements take a significant part of this area [4-6]. The interest in neural networks is due it helps to make a contribution to a better understanding of brain functionality, that also is of a great interest [7].

The investigation of multilayer networks is of interest because the networks of brain have a multilayer structure [8]. The studies help to understand how the brain works. In recent years, many researchers had done a lot of experimental studies of the brain [9-14]. For its investigation different methods for visualization and diagnostics are applied [15-18].

One of the important questions is the synchronization in the brain. Synchronization and nonlinear dynamics are investigated in many systems of different nature [19-23]. Phase synchronization between the neurons within the local ensemble results in the high amplitude of the electrical potential registered by the electrode on the sensor or source level.

In this paper we investigate the dynamics of complex network of Hodgkin-Huxley 2 neurons which consists of 3 sub-networks. The first small network receives external signal which is transferred into a spike sequence. Then it is transmitted to two networks interacting via an inhibitory coupling and working together to process the signal. We observe that the synchronization index in both networks periodically changes in time: the time intervals with the high SI alternate with the time intervals where synchronization index is low. We calculate correlation between them and find that when adjusting the strength of the inhibitory coupling one can observe SI in these networks changes either in phase or out of phase.

This work was supported by the Russian Foundation for Basic Research (Grants 18-32-20129 and 19-52-55001).

II. MODEL

The system under study represents the input network of $N^{ex} = 5$ and 2 networks of $N_1 = N_2 = 50$ Hodgkin-Huxley neurons. The external stimulus of constant current with amplitude A is applied to the N^{ex} network. The neurons inside it are connected to each other with the coupling strength chosen randomly from the range [0,0.15]. This network is connected to the two N_1 and N_2 networks by one-directional excitatory couplings with coupling strength $g_c = 0.05$ and probability $p = 30$ %. The networks N_1 and N_2 are connected to each other by two-directional inhibitory couplings with coupling strength g_c^{ex} and probability $p = 30$ %. Inside them the neurons are connected to each other according to "small-world" (SW) topology with coupling strength g_c^{in}.

The time evolution of the transmembrane potential of each HH neuron is given by [24]

$$C_m \frac{dV}{dt} = -g_{\mathrm{Na}} m^3 h (V - V_{\mathrm{Na}}) - g_{\mathrm{K}} n^4 (V - V_{\mathrm{K}}) - g_{\mathrm{L}} (V - V_{\mathrm{L}}) + I^{\mathrm{ex}} + I^{\mathrm{syn}} \quad (1)$$

where $C_m = 1\mu F/cm^3$ is the capacity of cell membrane, I^{ex} is an external bias current injected into a neuron, V is the membrane potential of a neuron, $g_{\mathrm{Na}} = 120\ mS/cm^2$, $g_{\mathrm{K}} = 136\ mS/cm^2$ and $g_{\mathrm{L}} = 0.3\ mS/cm^2$ receptively denote the maximal sodium, potassium and leakage conductance when all ion channels are open. $V_{\mathrm{Na}} = 50\ mV$, $V_{\mathrm{K}} = -77\ mV$ and $V_{\mathrm{L}} = -54.4\ mV$ are the reversal potentials for sodium, potassium, and leak channels respectively. m, n and h represent the mean ratios of the open gates of the specific ion channels. n^4 and m^3h are the mean portions of the open potassium and sodium ion channels within a membrane patch. The dynamics of gating variables (x=m,n,h) depending on rate functions $\alpha_x(V)$ and $\beta_x(V)$ are given:

$$\frac{dx}{dt} = \alpha_x(V)(1 - x) - \beta_x(V)x + \xi_x(t), \qquad x = m, n, h \quad (2)$$

$\xi_x(t)$ is independent zero mean Gaussian white noise sources.

I_i^{syn} is the total synaptic current received by i-th neuron. We consider coupling via chemical synapses. The synaptic current takes the form

$$I_i^{\mathrm{syn}} = \sum_{j \in \mathrm{neight}(i)} g_c e^{-(t - t_0^j)/\tau_{\mathrm{syn}}} (E_{\mathrm{rev}} - V_i) \quad (3)$$

where the alpha function $\alpha(t)$ describes the temporal evolution of the synaptic conductance, g_c is the maximal conductance of the synaptic channel, t_0^j is the time at which presynaptic neuron j fires, $\tau_{\mathrm{syn}} = 3$ ms.

III. RESULTS

We analyze neural dynamics of N_1 and N_2 networks. Excitatory coupling inside each network leads to

978-1-7281-7286-6/20/$31.00 ©2020 IEEE

synchronization of these neurons (first 50 neurons of N_1 network and second ones of N_2 neurons on Fig. 1(a)). Since networks are interconnected via an inhibitory coupling, depending on the coupling strength an anti-phase dynamics in the activities of them can be achieved. Black color illustrates the inhibition of neurons of one network by other neurons. Yellow color corresponds to spike generation, and one can see that thickness of yellow lines of each network changes through time which is connected to the synchronization of neurons.

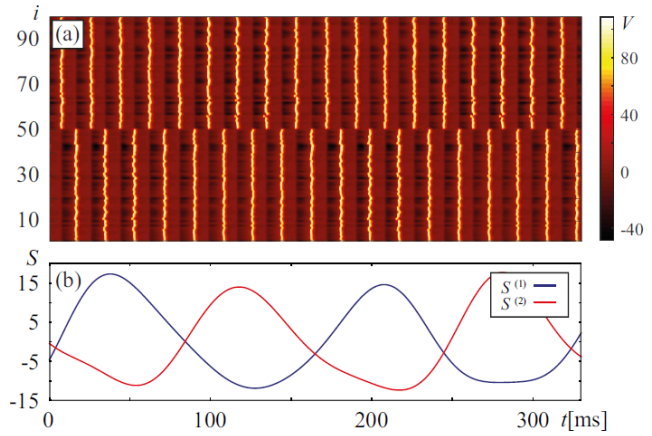

Fig. 1. (a) Time-space diagram of membrane potential V of the neurons of N_1 ($1 <= i <= 50$) and N_2 ($51 <= i <= 100$) networks. (b) Time evolution of synchronization indexes $S^{(1)}$ and $S^{(2)}$ for the same networks. Strength of coupling between N_1 and N_2 $g_c^{ex} = -0.1$, inside each one $g_c^{in} = 1.0$.

To investigate it we calculate synchronization index for networks N1 and N2 [25]:

$$S = \sqrt{\frac{1}{T}\sum_{n=1}^{T} \xi_n} \qquad (4)$$

were ξ_n is the standard deviation given as

$$\xi_n = \frac{1}{N}\sum_{i=1}^{N}\left(x_n^{(i)}\right)^2 - \left(\frac{1}{N}\sum_{i=1}^{N} x_n^{(i)}\right)^2 \qquad (5)$$

where T is a number of iterations, N is a number of neurons in the network. The smaller S, the better the synchronization; $S = 0$ means complete synchronization. We apply filtering in [0.004,0.015] Hz frequency band corresponding to the low-frequency modulation of macroscopic signal of each network. The examples of filtered synchronization indexes $S^{(1)}$ (N_1) and $S^{(2)}$ (N_2) are presented on Fig. 1(b). Low value of S means higher synchronization and vice versa. The thickness of yellow line and synchronization index are well correlated with each other: lower thickness means better synchronization, hence lower S.

Fig. 2 illustrates two-parametric diagram of synchronization indexes correlation versus g_c^{ex} and g_c^{in}. There is the light-yellow area of high positive correlation with width of 0.02 in the coupling strength between the networks g_c^{ex}. Boundary values of it go higher with decreasing coupling strength inside the networks g_c^{in}. So, to achieve maximal positive correlation both coupling strengths must be simultaneously changed in one direction (both decrease or both increase). This suggests that the excitatory current received by a neuron from the neurons of the same network and the inhibitory current received from another network should compensate each other. The same thing can be referred to achieving maximal anticorrelation dynamics (black areas on Fig. 2) when two networks demonstrate antiphase dynamics of the synchronization index oscillation. One can also note that the upper dark area of negative correlation on Fig. 2 ($-0.06 < g_c^{ex} < -0.015$) is the widest for $g_c^{in} = 1.0$ and becomes narrower with decreasing the last one.

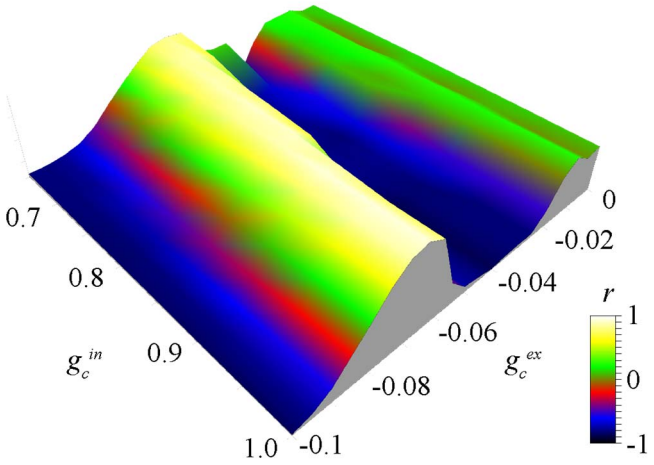

Fig. 2. Two-parametric diagram of correlation r of synchronisation indexes $S^{(1)}$ and $S^{(2)}$ versus coupling strength between the networks g_c^{ex} and coupling strength inside the networks g_c^{in}.

IV. Conclution

We have investigated the dynamics of complex network of Hodgkin-Huxley neurons. It consists of 3 sub-networks. The first small network Nex receives external signal which is transferred into a spike sequence. Then it is transmitted to two small-world N1 an N2 networks interacting via an inhibitory coupling and working together to process the signal. We have observed that the synchronization index in both networks periodically changes in time: the time intervals with the high SI alternate with the time intervals where SI is low. We have calculated correlation between them and found that when adjusting the strength of the inhibitory coupling one can observe SI in these networks changes either in phase or out of phase.

References

[1] V. V. Makarov, S. Kundu, D. V. Kirsanov, N. S. Frolov, V. A. Maksimenko, D. Ghosh, et.al., "Multiscale interaction promotes chimera states in complex networks," Communications in Nonlinear Science and Numerical Simulation, vol. 71, pp. 118-129, 2019.

[2] V. V. Makarov, M. O. Zhuravlev, A. E. Runnova, P. Protasov, V. A. Maksimenko, N. S. Frolov et.al., "Betweenness centrality in multiplex brain network during mental task evaluation," Phys. Rev. E, vol. 98, no. 6, pp. 062413, 2018.

[3] M. M. Danziger, O. I. Moskalenko, S. A. Kurkin, X. Zhang, S. Havlin, and S. Boccaletti, "Explosive synchronization coexists with classical synchronization in the Kuramoto model," Chaos: An Interdisciplinary Journal of Nonlinear Science, vol. 26, no. 6, p. 065307, 2016.

[4] A. V. Andreev, M. V. Ivanchenko, A. N. Pisarchik, and A. E. Hramov, "Stimulus classification using chimera-like states in a spiking neural network," Chaos, Solitons & Fractals, vol. 139, p. 110061, 2020.

[5] A. N. Pisarchik, V. A. Maksimenko, A. V. Andreev, N. S. Frolov, V. V. Makarov, M. O. Zhuravlev, et al. "Coherent resonance in the distributed cortical network during sensory information processing," Scientific Reports, vol. 9. no 1. pp. 1-9, 2019.

[6] A. Andreev, V. Makarov, A. Runnova, and A. Hramov, "Coherent resonance in neuron ensemble with electrical couplings," Cybernetics and Physics, vol. 6, no. 3, pp. 135-138, 2017.

[7] V. Maksimenko, V. Khorev, V. Grubov, A. Badarin, and A. E. Hramov, "Neural activity during maintaining a body balance," Proc. SPIE, vol. 11459, p. 1145903, 2020.

[8] V. V. Makarov, D. Kirsanov, M. Goremyko, A. Andreev, and A. E. Hramov, "Nonlinear dynamics of the complex multi-scale network," Proc. SPIE, vol. 10717, p.. 1071729, 2018.

[9] A. N. Pavlov, A. E. Runnova, V. A. Maksimenko, O. N. Pavlova, D. S. Grishina, and A. E. Hramov, "Detrended fluctuation analysis of EEG patterns associated with real and imaginary arm movements," Physica A: Statistical Mechanics and its Applications, vol. 509, pp. 777-782, 2018.

[10] V. Maksimenko, A. Badarin, V. Nedaivozov, D. Kirsanov, and A. Hramov, "Brain-computer interface on the basis of EEG system Encephalan," Proc. SPIE, vol. 10717, p. 107171R, 2018.

[11] A. A. Badarin, V. V. Skazkina, and V. V. Grubov, "Studying of human's mental state during visual information processing with combined eeg and fnirs," Proc. SPIE, vol. 11459, p. 114590D, 2020.

[12] V. A. Maksimenko, A. E. Hramov, V. V. Grubov, V. O. Nedaivozov, V. V. Makarov, and A. N. Pisarchik, "Nonlinear effect of biological feedback on brain attentional state," Nonlinear Dynamics, vol. 95, no. 3, pp. 1923-1939, 2019.

[13] P. Chholak, G. Niso, V. A. Maksimenko, S. A. Kurkin, N. S. Frolov, E. N. Pitsik, et al., "Visual and kinesthetic modes affect motor imagery classification in untrained subjects" Scientific reports, vol. 9, no. 1, pp. 1-12, 2019.

[14] P. Chholak, A. N. Pisarchik, S. A. Kurkin, V.A. Maksimenko, A.E. Hramov, "Phase-amplitude coupling between mu- and gamma-waves to carry motor commands," 2019 3rd School on Dynamics of Complex Networks and their Application in Intellectual Robotics (DCNAIR), IEEE, pp. 39-45, 2019.

[15] A. E. Hramov, V. Grubov, A. Badarin, V. A. Maksimenko, and A. N. Pisarchik, "Functional near-infrared spectroscopy for the classification of motor-related brain activity on the sensor-level," Sensors, vol. 20, no. 8, p. 2362, 2020.

[16] V. A. Maksimenko, S. A. Kurkin, E. N. Pitsik, V. Y. Musatov, A. E. Runnova, T. Y. Efremova, et.al., "Artificial neural network classification of motor-related eeg: An increase in classification accuracy by reducing signal complexity," Complexity, vol. 2018, 2018.

[17] S. Kurkin, P. Chholak, V. Maksimenko, and A. Pisarchik, "Machine learning approaches for classification of imaginary movement type by meg data for neurorehabilitation," 2019 3rd School on Dynamics of Complex Networks and their Application in Intellectual Robotics (DCNAIR), IEEE, pp. 106-108, 2019.

[18] N. S. Frolov, S. A. Kurkin, M. V. Khramova, A. A. Badarin, A. A. Koronovskii, A. N. Pavlov, and A. E. Hramov, "Perspective sub-THz powerful microwave generator" nanovircator" for T-rays biomedical diagnostics," Pros. SPIE, vol. 9917, p. 991721, 2016.

[19] A. A. Badarin, S. A. Kurkin, A. A. Koronovskii, A. E. Hramov, and A. O. Rak, "Processes of virtual cathodes interaction in multibeam system," Physics of Plasmas, vol. 25, no. 8, p. 083110, 2018.

[20] Sadovnikov A. V., Grachev A. A., Odintsov S. A., Sheshukova S. E., Sharaevskii Y. P., Nikitov S. A., "Spin-wave transport along in-plane magnetized laterally coupled magnonic stripes," IEEE Magnetics Letters. vol. 8, pp. 1-4, 2017.

[21] A. Badarin, S. Kurkin, and A. Hramov, "Multistability in a relativistic electron beam with an overcritical current," Bulletin of the Russian Academy of Sciences: Physics, vol. 79, no. 12, pp. 1439–1442, 2015.

[22] Sadovnikov A. V., Grachev A. A., Beginin E. N., Odintsov S. A., Sheshukova S. E., Sharaevskii Y. P., et al. "Spatial dynamics of hybrid electromagnetic spin waves in a lateral multiferroic microwaveguide," JETP Letters, vol. 105, no. 6, pp. 364-369, 2017.

[23] A. A. Koronovskii, O. I. Moskalenko, A. S. Pavlov, N. S. Frolov, and A. E. Hramov, "Generalized synchronization in the action of a chaotic signal on a periodic system," Technical Physics, vol. 59, no. 5, pp. 629-636, 2014.

[24] A. L. Hodgkin, and A. F. Huxley, "A quantitative description of membrane current and its application to conduction and excitation in nerve," The Journal of physiology, vol. 117, no. 4, p. 500, 1952.

[25] J. M. Sausedo-Solorio, and A. N. Pisarchik, "Synchronization in network motifs of delay-coupled map-based neurons," The European Physical Journal Special Topics, vol. 226, no. 9, pp. 1911-1920, 2017.