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Interaction of Chimera States in a Multilayered Network of Nonlocally Coupled Oscillators

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Abstract—The processes of formation and evolution of chimera states in the model of a multilayered network of nonlinear elements with complex coupling topology are studied. A two-layered network of nonlocally intralayer-coupled Kuramoto–Sakaguchi phase oscillators is taken as the object of investigation. Different modes implemented in this system upon variation of the degree of interlayer interaction are demonstrated.

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At present, considerable interest in the investigation of chimera states [1] implemented in ensembles of nonlinear oscillators of different nature is observed in radio physics, biophysics, and nonlinear dynamics. Such states, which are characterized by coexistence of groups of coherent and incoherent elements in networks of coupled oscillators, were first discovered in 2002 [2] in a network of nonlocally coupled nonlinear elements described by a Ginzburg–Landau-type equation.

A number of papers devoted to implementation of chimera states in networks of nonlinear elements characterized by dynamics of different types have been published recently. The formation of chimera states in both one-dimensional systems (chains of coupled oscillators) [3] and distributed systems consisting of equidistant oscillators that are described by the Rössler [4], FitzHugh-Nagumo [5], etc., equations have been considered in this context. Chimera states were discovered in networks with different coupling topology. In particular, chimera states were in [6] obtained for the cases of global, nonlocal, and local coupling as exemplified by a Hindmarsh–Rose neural network. In [7], along with symmetric couplings, the possibility of implementation of chimera states in a scale-free network was demonstrated. The interest in chimera states, along with analysis of model systems, is also due to by the detection of states with properties that correspond to chimeras in real systems. Such systems, as a rule, include objects of biological, chemical, and electronic nature [8–10].

The problem of stability of chimera states [11], including the case of interaction of a system exhibiting a chimera with a system characterized by coherent behavior of all nodes or complete incoherence, is an important and hardly studied problem among the large number of effects associated with the formation of chimera states. Obviously, this state of affairs is widespread in real systems related to corresponding fields of science (for example, neuroscience [12, 13]), and it is investigated, along with because of theoretical interest, due to the promising character of practical application for understanding of the processes in neuron networks [14].

The most appropriate mathematical and physical model for investigation of the processes of interaction of objects with a network structure is a multilayered network. In this model, each element is characterized by two types of couplings. The first type characterizes the element's interaction with other nodes of the network within one layer. The second layer determines the coupling of a given element with elements related to other layers of the network. Depending on the specific problem, the coupling configurations between elements of the multilayered network can be quite different. In the framework of this Letter, we consider the configuration described in [15].

According to [15], a network consisting of N^M elements is represented as a set of M layers (N elements in each layer). The couplings between elements inside the layer are nonlocally distributed [6] (each element is coupled with $2R$ neighboring elements), and interaction between the layers is implemented via local couplings between the adjacent elements. This model is schematically shown in Fig. 1a. Quantities ϕ_i^j corre-

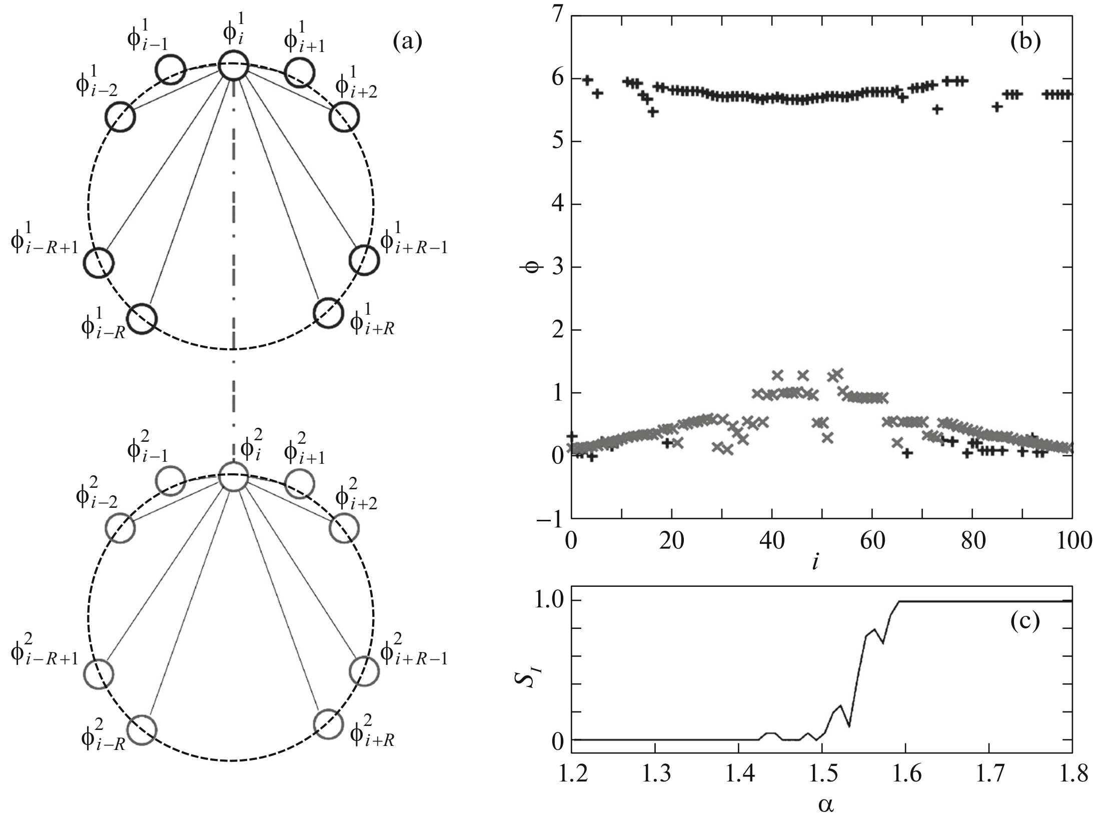

Fig. 1. (a) Schematic diagram of the model of a multilayered network with nonlocal coupling between elements of one layer, (b) oscillator-phase distributions in the layers in the case of absence of a multilayered coupling, and (c) strength of incoherence S_I as a function of the phase shift α. Oscillators of the first layer are denoted by "+," and oscillators of the second layer are denoted by "×."

spond to dynamic variables characterizing the state of the network node, and indices i and j correspond to the element number inside the layer and the layer number. Solid lines denote couplings of the element ϕ_i^j with the neighboring elements inside the layer, and the dash-dotted line shows the interlayer coupling implemented via interaction of first layer element ϕ_i^1 with neighboring second layer element ϕ_i^2.

For simulating the dynamics of the network node, here we use a Kuramoto–Sakaguchi phase oscillator [16], which is often employed as a basic model for numerical and analytical study of chimera states [17],

$$\frac{d\phi_i^j}{dt} = \omega_i^j - \frac{\lambda_1}{2R+1}\sum_{k=i-2R}^{i+2R}\sin(\phi_i^j - \phi_k^j + \alpha) + \frac{\lambda_2}{M}\sum_{l\neq j}\sin(\phi_i^j - \phi_i^l). \tag{1}$$

Here, ω_i^j is the oscillator eigenfrequency, λ_1 is the coupling coefficient between oscillators inside the layer, λ_2 is the interlayer coupling coefficient, R is the coupling radius, M is the number of layers, and α is the constant phase shift. In this Letter, we consider the network of identical oscillators $\omega_i^j = 1$, $\forall i$, j. The parameter values are chosen as follows: $\lambda_1 = 0.085$, $N = 100$, $M = 2$, and $R = 35$. It should be noted that the oscillator characteristics and the coupling topology inside the studied layers of the network are similar and satisfy the condition of chimera-state formation [17].

For implementation of qualitatively different states in the network layers, we use different configurations of initial phase distributions $\phi_i^1(0) \neq \phi_i^2(0)$, $\forall i \in [1, N]$. For the applied initial conditions in the case in which there is no coupling between the layers, the corresponding states are shown in Fig. 1b. It can be seen that, in both the first and second layers, some of the oscillators are in the coherent state, while the others are incoherent. It can be easily seen that the numbers of oscillators in coherent and incoherent clusters in different layers do not correspond with each other.

For investigation of the processes of interaction between these chimera states, we perform numerical simulation of the system dynamics with increasing interlayer coupling parameter λ_2. For qualitative diag-

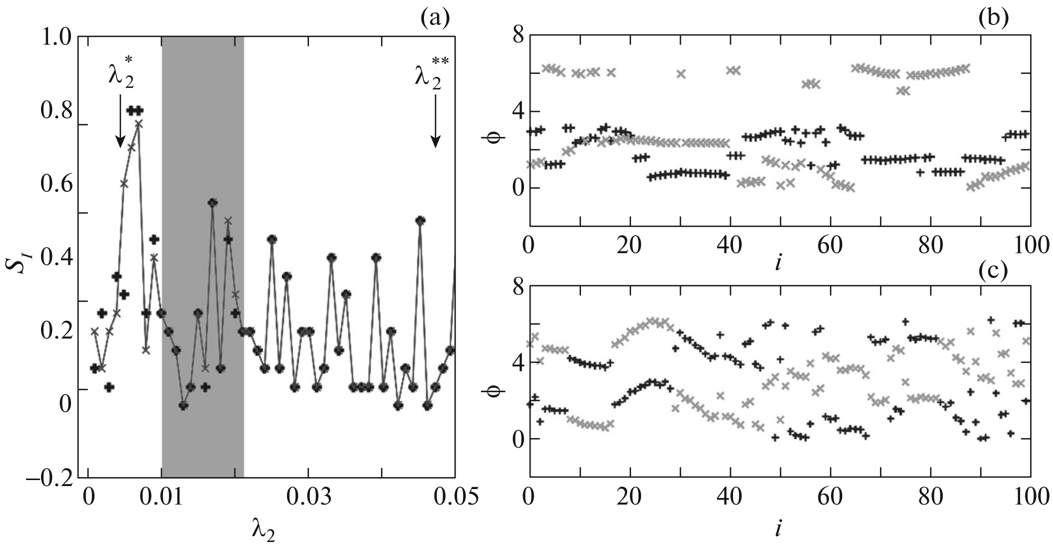

Fig. 2. (a) Strength of incoherence S_I as a function of λ_2 calculated for two layers, distributions of oscillator phases in the layers for (b) $\lambda_2 = \lambda_2^*$ and (c) $\lambda_2 = \lambda_2^{**}$. The positions of points $\lambda_2 = \lambda_2^*$ and $\lambda_2 = \lambda_2^{**}$ are shown by arrows in (a).

nosis of the chimera state, we calculate S_I (2), which characterizes the strength of incoherence. For calculation of this quantity, the studied ensemble of oscillators is divided into m groups of elements, with n elements in each group. For the given ensemble, S_I is determined as follows:

$$S_I = 1 - \frac{\sum_{r=1}^{m} \Theta(\delta - \sigma_r)}{m}, \tag{2}$$

where $\Theta(\cdot)$ is the Heaviside function, σ describes standard deviation (3) characterizing oscillators in the group with index r, and $\delta = 0.035$ is the threshold value. Quantity σ_r is calculated for each group using the relation

$$\sigma_r = \left\langle \sqrt{\frac{1}{n} \sum_{s=n(r-1)+1}^{m} [\varphi_s - \Phi]^2} \right\rangle, \tag{3}$$

where $\langle\cdot\rangle$ denotes averaging over the time interval and Φ is the average phase over the ensemble.

Depending on the value of S_I, the network state can be interpreted as completely coherent, $S_I = 0$; completely incoherent, $S_I = 1$; or a chimera $0 < S_I < 1$ corresponding to coexistence of the coherent and incoherent clusters.

Figure 1c shows S_I as a function of α characterizing the phase relation in the Kuramoto–Sakaguchi model. The parameter domain corresponding to chimera states is shaded. Taking into account this dependence, here the value of parameter α for each layer was taken equal to $\alpha = 1.45$.

Figure 2a shows parameter S_I calculated for two layers of the considered network as a function of the growing parameter λ_2. It can be seen that, for $\lambda_2 < 0.01$, chimera states with different properties are implemented. The typical oscillator phase distributions for this case are shown for two layers in Fig. 2b. It can be seen that there exist coherent and incoherent clusters in both the first and second layers. However, the phase relations inside these clusters and, moreover, the number of elements included in them differ in different layers of the network. If the value of λ_2 is in an interval [0.01, 0.02], both states characterized by similar and different oscillator phase distributions in the layers can be implemented in the network. With further growth of λ_2, a transition to synchronous-network layer dynamics with formation of identical chimera states in the layers is observed. The oscillator phase distributions for this case are shown in Fig. 2c. It can be seen that the elements in the layers demonstrate similar dynamics.

Thus, here, the interaction of chimera states in the multilayered network of phase oscillators was studied for the first time. It was demonstrated that the growth of the degree of interlayer interaction does not result in destruction of chimera states. Moreover, it was discovered that there exist ranges of the degree of interlayer interaction in which implementation of both different and similar chimera states in the network layers is possible.

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