Image denoising with the dual-tree complex wavelet transform

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ABSTRACT

The purpose of this study is to compare image denoising techniques based on real and complex wavelet-transforms. Possibilities provided by the classical discrete wavelet transform (DWT) with hard and soft thresholding are considered, and influences of the wavelet basis and image resizing are discussed. The quality of image denoising for the standard 2-D DWT and the dual-tree complex wavelet transform (DT-CWT) is studied. It is shown that DT-CWT outperforms 2-D DWT at the appropriate selection of the threshold level.

Keywords: wavelet-transform, image denoising, thresholding, optical images

1. INTRODUCTION

Images are often corrupted by noise during transmission or acquisition, and its removal is an important problem in many practical applications. At present, wavelet-based denoising techniques are widely used to improve the quality of corrupted images.[1–5] Such techniques assume a decomposition of an image over a basis of specific soliton-like functions with further correction of the decomposition coefficients. Besides denoising, transition into the wavelet space provides a possibility of image compression[6,7] and this approach is applied, e.g., in computer graphics within the format JPEG2000. Generally, wavelet denoising outperforms Fourier-based filtering since it is a more appropriate tool when dealing with localized features of signals and images.

Idea underlying wavelet denoising consists in the separation of image features at different scales. Within the multiresolution analysis based on the discrete wavelet transform (DWT) with the octave-band filterbank (FB),[8,9] large and small wavelet coefficients are more likely due to important image features and noise, respectively. If small coefficients are thresholded, main structure of the processed image remains unchanged. Thresholding represents a kind of nonlinear smoothing that strongly depends on the selection of threshold level l. For small l, only few wavelet coefficients are changed to zeros, and the reconstructed image may still be corrupted by noise. For large l, the performed thresholding can destroy image features. Optimal threshold selection is a subject of many studies.[2–6] Often, two main approaches are considered, namely, hard and soft thresholding.[1] The first approach assumes that only small wavelet-coefficients are corrected (changed to zeros) before image reconstruction in the course of inverse DWT. It is applied when the precise recovery of signal magnitude should be provided although the performed filtering produces spurious structures caused by discontinuities of the threshold function. The second approach does not keep the magnitude, but it can retain the regularity of signal and this circumstance is an important property widely used for image denoising. Other thresholding methods can also be applied.[10–13]

Appropriate selection of the wavelet basis and the thresholding parameters can significantly improve the quality of noise reduction. Classical DWT typically deals with the family of Daubechies wavelets. However, this

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technique has four main shortcomings:[14] 1) oscillations of the wavelet-coefficients around singularities complicating signal processing; 2) shift variance leading to unpredictable change in the wavelet coefficient patterns for shifted singularities; 3) aliasing that results in artifacts in the reconstructed signal after the wavelet coefficient processing; 4) lack of directional selectivity creating difficulties for analysis of image features. Aiming to improve the wavelet-based denoising, an approach known as the *dual-tree complex wavelet transform* (DT-CWT) was proposed that is nearly shift invariant and directionally selective in two and higher dimensions.[15–18] This approach considers complex-valued (analytic) wavelets constructed from real-valued functions by adding the imaginary part introduced via the Hilbert transform. The DT-CWT of a signal represents two real DWTs realized for the real and the imaginary part of the transform, respectively.

In this paper we perform a comparison of different wavelet-based denoising techniques. We analyze possibilities provided by the classical DWT with hard and soft thresholding and discuss how the selection of the wavelet basis influences the results of noise reduction. Due to shift variance of the standard DWT we expect that image resizing changes wavelet coefficient patterns and, therefore, minimal distortions of the filtered image can be realized for different wavelet bases being applied to the original and to the resized images. Finally, we compare denoising performances for the standard DWT and the DT-CWT.

2. 1-D AND 2-D DISCRETE WAVELET TRANSFORMS AND THRESHOLDING

Unlike the Fourier transform, DWT operates with localized functions representing dilated and translated versions of a "mother" wavelet $\psi(t)$. Most practical applications of the wavelet theory are based on the multiresolution analysis[8–10,19–21] that introduces a low-pass scaling function $\varphi(t)$ performing approximation of a signal at different levels of resolution with further analysis provided using wavelets. Decomposition of a signal $f(t)$ in terms of scaling functions and wavelets produces a series of coefficients carrying information about the signal's structure on independent scales

$$f(t) = \sum_k c_k \varphi(t-k) + \sum_j \sum_k d_{j,k} 2^{j/2} \psi(2^j t - k). \quad (1)$$

Analogous decomposition can be performed for another level of resolution of the considered signal.[8] A fast (pyramidal) algorithm[2] to estimate the decomposition coefficients c_k and $d_{j,k}$ based on analysis FBs is implemented as a recursive application of low-pass and high-pass filters with down-sampling by 2. The inverse transform performed with synthesis FBs and up-sampling operations provides perfect reconstruction of the signal. This approach is referred to as 1-D DWT and is widely applied to one-dimensional time series.

When dealing with images, a generalization of the given approach is considered, namely, 2-D DWT. It assumes implementation of 1-D analysis FBs to the columns and then to the rows of an image. This procedure produces four sub-images (one fourth of the original size): a lower resolution sub-image (LL) and three sub-images reflecting details related to horizontal (HL), vertical (LH) and diagonal (HH) structures. For an image of the size N_1 x N_2, e.g., application of 1-D DWT to the columns produces two sub-images of the size $N_1/2$ x N_2. Further application of 1-D DWT to each row leads to four sub-images of the size $N_1/2$ x $N_2/2$. At the next level of resolution this procedure is repeated for the LL sub-image. Such technique is referred to as the separable filtering. Unlike "true" non-separable 2-D FBs,[2,23] it is commonly used due to its simplicity. Perfect reconstruction of the original image from sub-images is obtained in the course of the inverse procedure realized with synthesis FBs.[4,7,10]

Wavelet-based image denoising is provided by correction of wavelet coefficients obtained after the implementation of analysis FBs at the stage before applying synthesis FBs. It is typically assumed that the analyzed image is corrupted by a low-intensity Gaussian noise. In this case, noise is presented at small scales and is associated with low-valued wavelet-coefficients while large coefficients carry information about important image features. An obvious approach for image denoising is to remove (change to zeros) noise-related coefficients without destroying image structures. This approach is realized by thresholding of wavelet-coefficients at small levels of resolution. Since these coefficients are related to image features at small scales, their removal is equivalent to some kind of nonlinear smoothing. Selection of threshold parameters clearly influences the quality of image denoising.

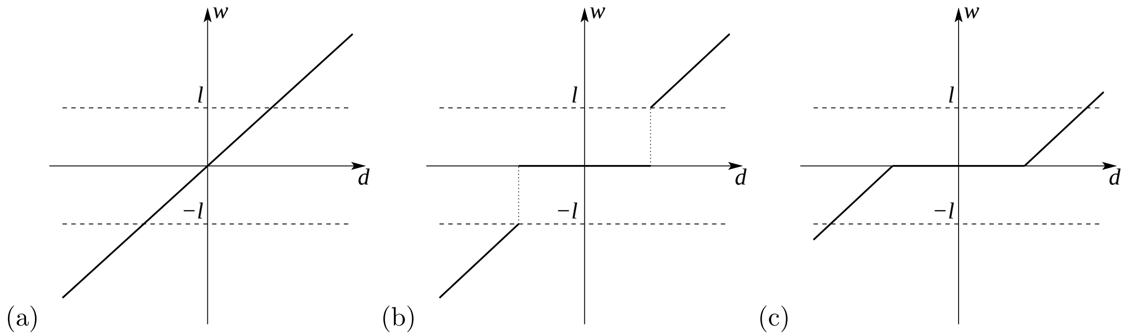


Figure 1. Threshold function associated with (a) unfiltered data, (b) hard threshold approach, and (c) soft threshold approach.

Two main approaches are applied for thresholding (Fig. 1b,c). If the threshold function is selected as $w(d) = d$ (Fig. 1a), the coefficients remains unchanged and the perfect image reconstruction is obtained. Hard thresholding approach (Fig. 1b) assumes that only low-valued wavelet-coefficients are changed to zeros:

$$w(d) = \begin{cases} d, & \mid d \mid \geq l, \\ 0, & \mid d \mid < l. \end{cases} \tag{2}$$

Large (most important) coefficients remain unchanged and, therefore, main image features are kept during this procedure. Disadvantage of hard thresholding is the presence of discontinuities of the function (2). These discontinuities in the wavelet domain produce spurious structures in the reconstructed image. In order to avoid them, soft thresholding approach is widely applied. Within this approach, threshold function is selected as

$$w(d) = \begin{cases} d - l, & d \geq l, \\ d + l, & d \leq -l, \\ 0, & \mid d \mid < l. \end{cases} \tag{3}$$

Because all coefficients are corrected, signal magnitude may change after the reconstruction stage. For image denoising, however, it is typically more important to keep signal regularity than to reproduce exact magnitudes. Due to this, soft thresholding is a more preferable approach for wavelet denoising.[3]

3. DUAL-TREE COMPLEX WAVELET TRANSFORM

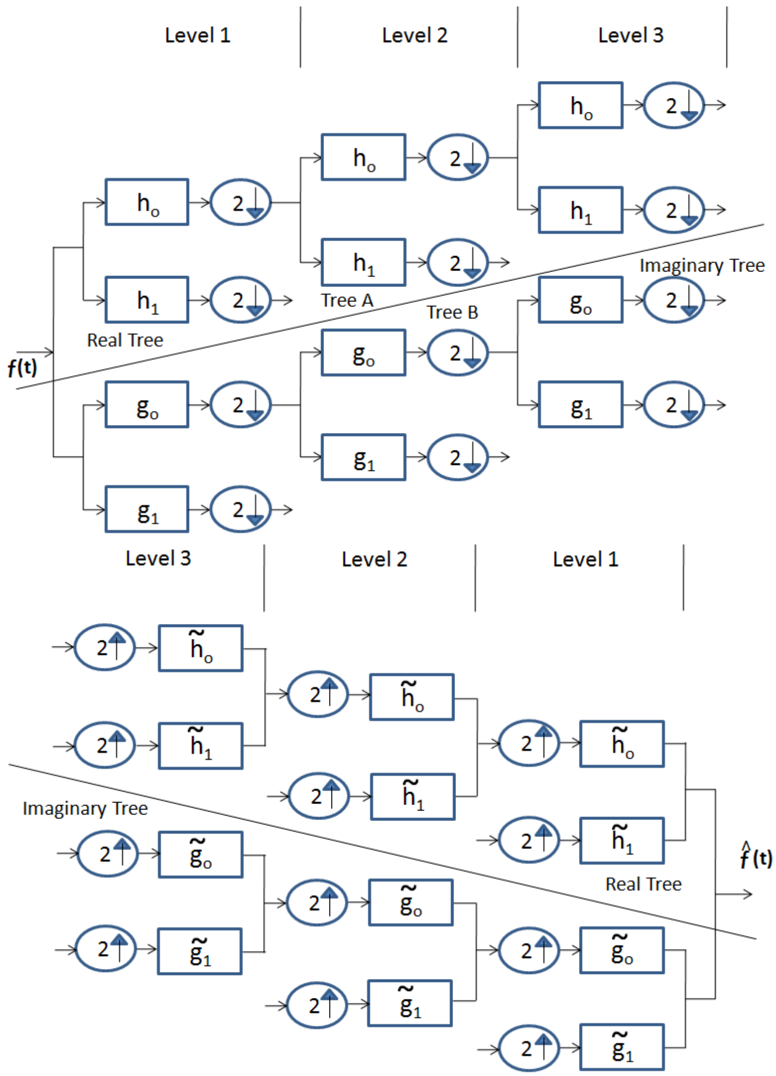
As it was already mentioned in Introduction, standard DWT has several shortcomings among which shift sensitivity and poor directionality are the most critical. Besides, it does not provide phase information that is also important for some applications. Aiming to improve abilities of this technique, an approach based on complex wavelet transform (CWT) was proposed.[15–18] Its general idea is to combine real-valued scaling function and wavelet with imaginary parts constructed via the Hilbert transforms,[22] providing complex-valued (analytic) low-pass and high-pass filters. In particular, complex wavelets $\psi^c(t)$ used within CWT can be written as

$$\psi^c(t) = \psi^r(t) + j\psi^i(t), \tag{4}$$

where the functions $\psi^r(t)$ and $\psi^i(t)$ individually form orthonormal bases.[14] At the real signal analysis wavelet transform is computed separately using the functions $\psi^r(t)$ and $\psi^i(t)$, thus producing complex wavelet coefficients

$$d^c_{jk} = d^r_{jk} + jd^i_{jk}. \tag{5}$$

Based on d^c_{jk}, magnitudes and phases can easily be introduced.[14] Because real and imaginary parts of d^c_{jk} are applied separately, the computational process consists of two parts and is realized within the approach called as the *dual-tree complex wavelet transform*.[15,16] At the analysis of one-dimensional signals this approach can be realized as two independent applications of 1-D DWT, with the functions $\psi^r(t)$ and $\psi^i(t)$, respectively. Algorithmically, this analysis is performed using two trees (Fig. 2a) and similar two trees procedure is realized



(a)

(b)

Figure 2. Analysis (a) and synthesis (b) FBs for 1-D DT-CWT.

for synthesis (Fig. 2b). In Figure 2, application of analysis and synthesis FBs is illustrated for three levels, and estimation of the wavelet-coefficients d^r_{jk} and d^i_{jk} can be performed in parallel. The first tree used to estimate d^r_{jk} is called as a real tree, while the second tree applied to compute d^i_{jk} is referred to as an imaginary tree. FBs used for analysis are constructed from mirror filters $(h_0,\ h_1)$ for a real tree, and $(g_0,\ g_1)$ for an imaginary tree with h_0 and g_0 being the low-pass filters, and h_1 and g_1 denoting the high-pass filters, respectively. Inverse transform is provided with synthesis FBs constructed from filters $(\tilde{h}_0,\ \tilde{h}_1)$, $(\tilde{g}_0,\ \tilde{g}_1)$.

Let us note, however, that DT-CWT has some features. For wavelets of compact support application of the Hilbert transform does not provide analytic properties of $\psi^c(t)$, and special design of FBs should be used to get scaling function and wavelet that are nearly analytic.[14] Here we apply filters[15] and their Matlab implementation developed by the group of I. Selesnick (http://eeweb.poly.edu/iselesni/WaveletSoftware/).

2-D DT-CWT assumes an extension of conjugate filtering. Thus, if $(h_x + jg_x)$ are the conjugate filters for the first dimension (x) and $(h_y + jg_y)$ are the conjugate filters for the second dimension (y) then the filters applied for 2-D case can be expressed as:

$$(h_x + jg_x)(h_y + jg_y) = (h_x h_y - g_x g_y) + j(h_x g_y - g_x h_y). \tag{6}$$

The FBs structure of 2-D DT-CWT is shown in Figure 3. It requires four trees for analysis (trees A, B, C and D) and four trees for synthesis (trees $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{D}$). An example of analysis FB for one tree is shown in Figure 4.

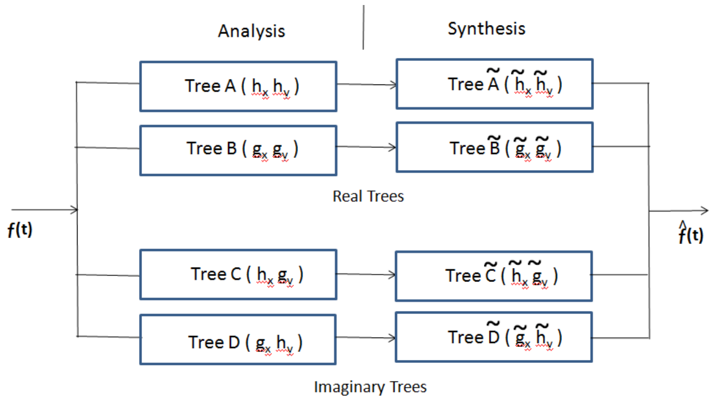


Figure 3. Filterbank structure of 2-D DT-CWT.

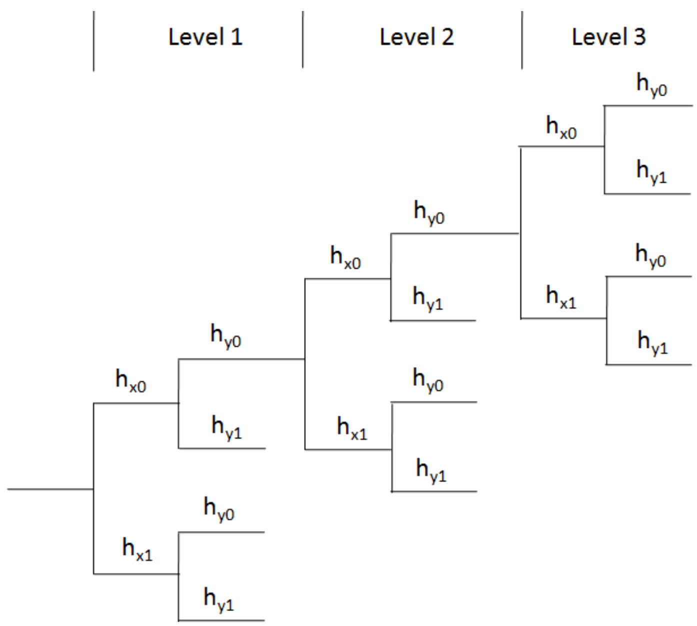


Figure 4. Filterbank structure of tree A (see Fig. 3).

4. RESULTS

Aiming to compare the quality of filtering provided by different wavelet-based approaches, a normally distributed random process was added to a selected image. Filtering with different wavelet bases, types of thresholding and threshold levels was further performed, and the root mean square (RMS) error between original and denoised images was estimated. Best noise reduction was associated with the minimal RMS error.

At the first stage, we compared the results for the classical 2-D DWT with Daubechies wavelets (D^2–D^{20}) and two types of thresholding (hard and soft thresholding; Fig. 1b,c) for different noise intensity. Figure 5 illustrates an example of the considered grayscale X-ray image of the tooth (original image, noisy image and filtered images with hard and soft thresholding, respectively).

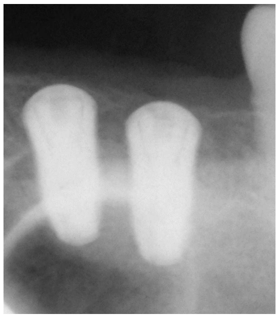
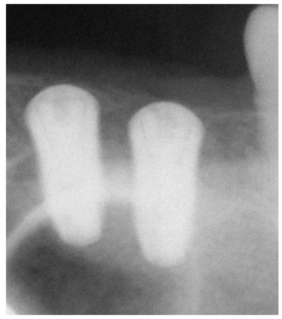
(a)  (b)  (c)  (d) 

Figure 5. An example of the wavelet-based denoising with 2-D DWT using Daubechies wavelet D^{10}: a) original image, b) noisy image (noise intensity 0.002), c) denoised image with hard thresholding approach, d) denoised image with soft thresholding approach.

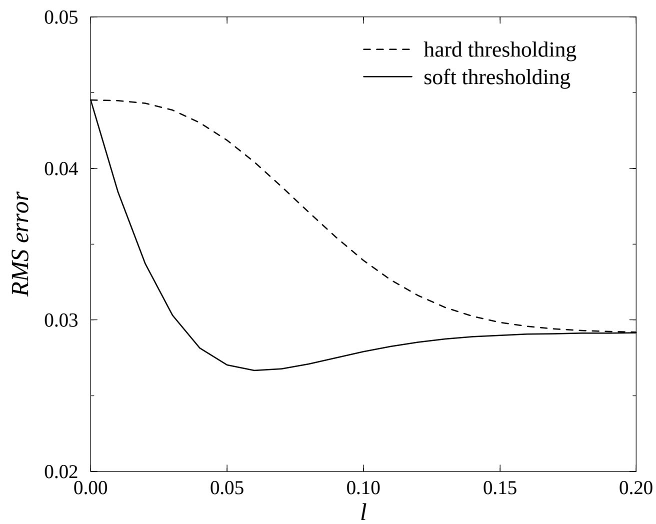


Figure 6. The dependence of RMS error vs. the threshold level for Daubechies wavelet D^{10} for hard and soft thresholding with optimal $l = 0.2$ and $l = 0.06$, respectively.

The value of RMS error depends on the selected wavelet basis. For the considered image (Figure 5), the minimal RMS error is reached for the basis D^{10} of Daubechies wavelets. By varying the value l, an optimal threshold associated with minimal RMS error can be selected (Figure 6).

It is important to note that optimal wavelet basis changes at image resizing. This circumstance is caused by the shift variance of the classical 2-D DWT with real bases leading to essential distinctions of wavelet coefficient patterns for shifted singularities. Thus, in particular, the minimal RMS error for the original and 1.1-fold scaled images are reached for distinct basis functions (D^{10} and D^7, respectively). Scaling provided with different resize factors influence the value RMS error, and the selection of the basis becomes an important problem.

According to Figure 6, selection of the threshold l is also important for image denoising. For optimal l, the soft thresholding approach provides significant reduction of RMS error compared with the hard thresholding. If l is selected quite large, then the quality of filtering with the soft thresholding is reduced, and the hard thresholding approach can provide better results. This circumstance is caused by the fact that the soft thresholding technique assumes corrections of all wavelet coefficients while large (most informative) wavelet-coefficients remain unchanged at the hard thresholding.

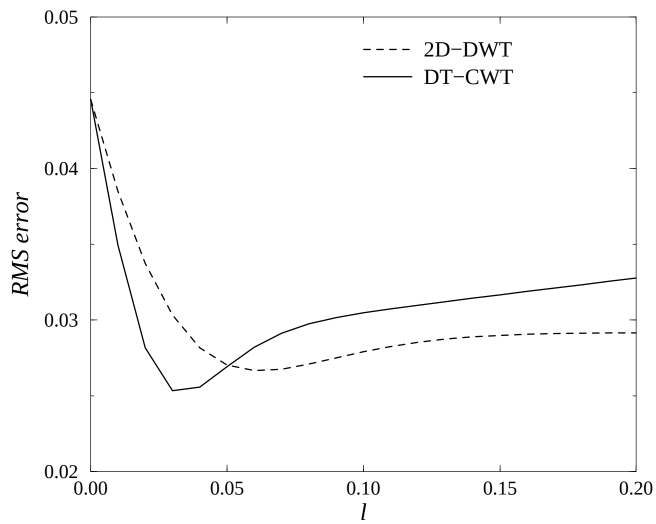


Figure 7. The dependence of RMS error vs. the threshold level for 2-D DWT and DT-CWT.

Reaching minimal RMS error with the standard DWT, at the second stage we compared the results with the DT-CWT approach. Using wavelet filters[15] we performed analogous estimations of RMS error for the soft thresholding and different threshold levels. The obtained results are illustrated in Figure 7 and certify better noise reduction of DT-CWT compared with the standard 2-D DWT, where the minimal RMS error of the standard 2-D DWT (0.0251) is larger than the minimal RMS error of the DT-CWT (0.0215). Similar results are obtained for other images, thus certifying better noise reduction performance of the complex wavelet transform.

5. CONCLUSION

In this paper we studied the possibility of improving the quality of image denoising with wavelet-based techniques. For this purpose, approaches based on the classical DWT and complex wavelet transform were considered. Within the first approach, Daubechies wavelets with hard and soft thresholding were used. It was shown that the soft thresholding reduces RMS error and, therefore, increases denoising performances of 2-D DWT compared with the hard thresholding. Nevertheless, the selection of the threshold level l becomes of high importance: for large l image denoising with the soft thresholding becomes less effective, and RMS error of the hard thresholding technique takes smaller values. Performing image resizing it was established that optimal wavelet basis (i.e., leading to minimal RMS error) depends on the image size. The latter circumstance is a consequence of the shift variance property being one of the main shortcomings of 2-D DWT. Statistical analysis performed for different images allowed us to reach better noise reduction with the soft thresholding approach.

Further, denoising abilities of DT-CWT were studied that is nearly shift invariant method. The latter means that image resizing does not essentially influence the wavelet coefficient patterns. Besides, this approach is directionally selective in two and higher dimensions, thus allowing keeping most important image structures if it is required to provide, e.g., image rotation. Within DT-CWT, real and imaginary parts of the complex-valued wavelets are related via the Hilbert transform, and application of analytic functions within the denoising technique leads to several advantages of the complex wavelet transform. The performed analysis has shown that DT-CWT reduces errors of image denoising compared with the classical 2-D DWT. However, selection of the threshold level also has an essential influence: inappropriate choice of l increases RMS error, and it becomes larger than RMS error of 2-D DWT. In general, it is preferable to start image denoising from small l, and to perform visual inspection of the filtered image with the increased threshold level. In this case image denoising with DT-CWT outperforms the standard DWT-based methods.

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