

Chaos and hyperchaos in the chain of quantum coherent elements

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Abstract—We study the chain of coupled quantum coherent systems with energy pumping and dissipation. Examples of such systems include the coupled Rydberg atoms with laser driving and spontaneous emission or electromagnetically driven artificial qubits with decoherence and dissipation. We found out that such system is able to demonstrate spontaneous onset of chaotic and even hyperchaotic oscillations, which are characterized by one or several positive Lyapunov exponents. Remarkably, the number of the positive Lyapunov exponents grows with the number of the chain elements. Hence a large chain is able to demonstrate highly irregular behavior. We investigate transition from regular to chaotic behavior and reveal the related instabilities.

Keywords—Rydberg atoms, chaos, hyperchaos, nonlinear dynamics

Recent progress in experimental techniques for the fabrication, control and measurement of quantum systems enables the routine creation of moderate-to-large arrays of coherent quantum elements, which allows some control of their quantum state. These systems are promising for numerous applications from quantum sensing and communication to universal quantum computing. Nevertheless, there is still little understanding of the associated emergent phenomena, which arise as the size of the system is increased and which critically affect the behavior of the system and our ability to control it.

To address this question, here we theoretically study how the dynamics of a chain of generic quantum coherent elements (e.g., superconducting qubits or Rydberg atoms), which interact with an external field, change with the number of chain elements. We find that even a small system (with just two-four quantum elements) can demonstrate chaotic behaviour. Remarkably, for systems with five or more elements we see the emergence of a phenomenon known as hyperchaos: a dynamical regime characterized by two or more positive Lyapunov exponents. Moreover, the number of positive Lyapunov exponents, i.e. the complexity of the hyperchaos, increases systematically, and hence controllably, with the number of qubits in the system.

The highly randomized nature of the hyperchaos that we identify shows similarities with thermalization, even though the system is far from equilibrium. Our results thus give unexpected insights into the dynamics of large artificial quantum coherent structures, which will be important for the design and control of quantum systems such as Rydberg molecules, quantum processors, simulators and detectors. In particular, we predict a highly random behavior even in systems comprising few quantum elements. The results also suggest a controllable way of switching between different dynamical regimes via the regular-to-chaos transitions. The latter is urgently needed in the topical emerging fields of quantum random generators and quantum chaotic cryptography.