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ABSTRACT

In this paper, we study the complex multi-scale network of nonlocally coupled oscillators for the appearance of chimera states. Chimera is a special state in which, in addition to the asynchronous cluster, there are also completely synchronous parts in the system. We show that the increase of nodes in subgroups leads to the destruction of the synchronous interaction within the common ring and to the narrowing of the chimera region.

Keywords: Chimera state, multilayered network, Hindmarsh-Rose neuron system

1. INTRODUCTION

Since the discovery of chimera states in 2002 by Kuramoto and Battogtokh, which are spatiotemporal models in which coherence and incoherence coexist,[1–4] these phenomena have already been reproduced repeatedly in the laboratory, which has provoked a flurry of interest in their properties. In this context, the formation of the chimera and chimera-like states was considered in one-dimensional systems (chains of coupled oscillators), as well as in distributed systems consisting of equidistantly arranged Rossler,[6] Fitzhugh-Nagumo oscillators,[7] etc.

Also, since that moment systems of various topologies have been investigated. In particular, in[8] the chimera states were demonstrated to exist in the network of Hindmarsh-Rose neurons in the case of global, non-local, and, even, the local coupling.

The traditional model for studying chimera states was the ring of nonlocal coupled oscillators,[9] but at the moment it has been well studied. Researchers bring new properties to this model. For example, in the article[10] the researhers study chimera states in networks of Van der Pol oscillators with hierarchical topology, by the stepwise transition from a nonlocal to a hierarchical topology. In the paper[11] authors consider the simplest network of coupled non-identical phase oscillators, and removing connections within the network in a random but systematically specified way, examining the boundaries of stability of chimera state. In paper[12] the model of identical phase oscillators arranged in networks of complex topology was investigated. Authors have found the existence of chimera states in which identical oscillators evolve into distinct coherent and incoherent groups, the coherent group of chimera states always contains the same oscillators no matter what the initial conditions are. The properties of chimera states and their dependence on parameters was investigated on both scale-free networks and Erdos-Renyi networks.

Also, studies are carried out to identify chimera states on topologies similar to different three-dimensional geometric figures. In the paper[13] authors studied coupled oscillators on the surface of a sphere, a system where two distinct classes of chimera states have been shown to exist: "spots" and "spirals". There is also a study to identify chimeras on the torus,[14] where was used asymptotic methods to derive the conditions under which two-dimensional chimeras can appear in a periodic space. Also was used numerical integration to explore the dynamics of these chimeras and determine which are dynamically stable. Usually chimera states are observed in systems with finite (and small) numbers of oscillators. In the paper,[15] focusing on networks of phase oscillators that are organized in two groups, was found that chimera states, corresponding to attracting periodic orbits, appear with as few as two oscillators per group. These findings suggest that chimeras, which bear striking

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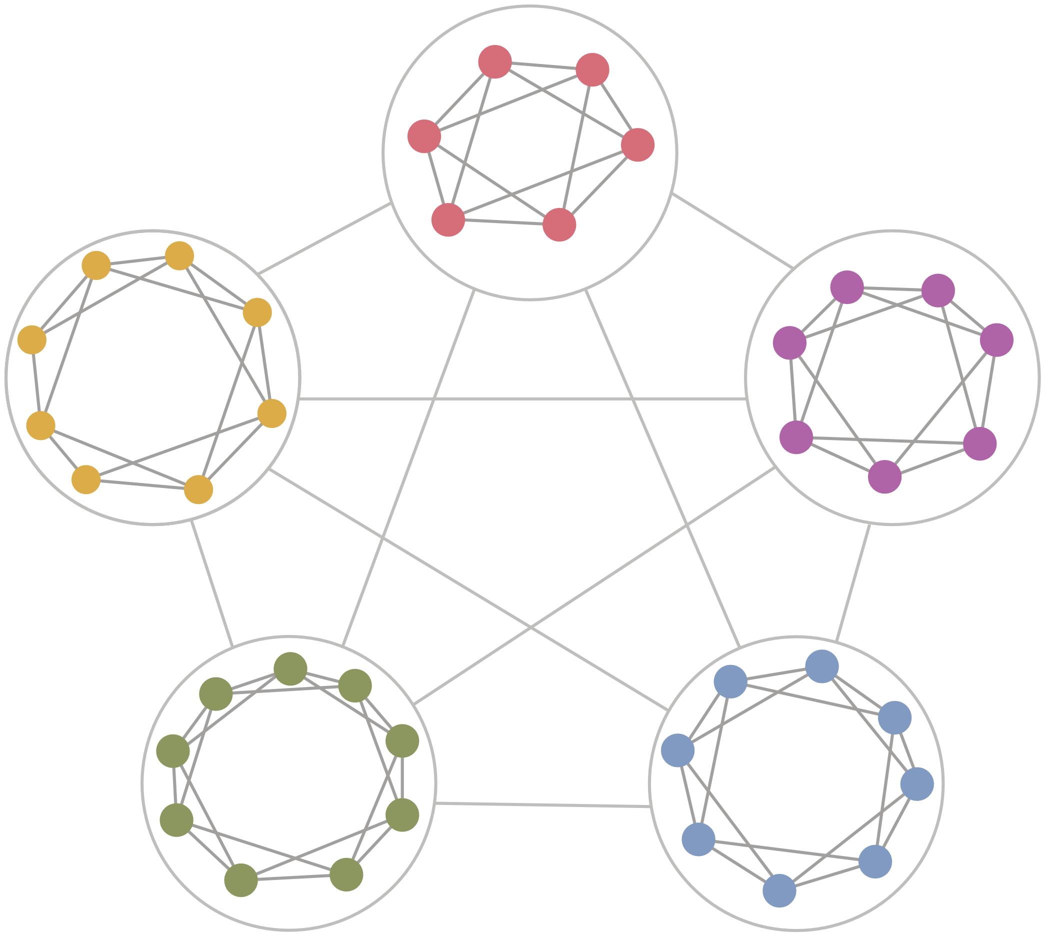

Figure 1. The principal structure of the multiscale network under study.

similarities to dynamical patterns in nature, are observable and robust in small networks that are relevant to a variety of real-world systems. Paper[16] shows that chimera states can be realized in experiments using a liquid-crystal spatial light modulator to achieve optical nonlinearity in a spatially extended iterated map system. In[17] was found the emergence of multicluster chimera states.

One of the interesting and specific topologies, which are reveal themself in many real-world systems as brain[18] are multilayer and/or multiplex networks.[19–21] From the dynamical perspective, the multilayer formulation has been applied both to networks whose layers coexist or alternate in time.[21] In both cases, the multilayer formulation allows to identify synchronization regions that arise as a consequence of the interplay between the layers topologies,[22–24] as well as to define new types of synchronization based on the coordination between layers.[25,29] Lately it is also shown, that the multiplex interaction between two non-locally coupled networks can lead to the excitation of chimera state in the parameter region, where it is not present in the absence of inter-layer coupling.[26–28]

In this paper, we study the chimera state in the network of complex topology, reperesenting the multiscale structure of nonlocally coupled oscillators. In particular, we examine how the emergence of the additional local scale-interaction effects the appearance of chimera states. We show that the increase of nodes in subgroups leads to the destruction of the synchronous interaction within the common ring and to the narrowing of the chimera region.

2. NUMERICAL MODEL

The system under study represents the M subgroups of oscillators, which are non-locally connected through common non-locally coupled ring. Each subgroup is formed by of N non-locally coupled oscillators. This structure is shown in Fig. **??**. Here one can see the multiscale structure, each subgroup with coupling radius R_N is a part of the common circle, which coupling radius is R_M. All networking nodes are represented by the Kuramoto-Sakaguchi (KS) phase model, $\varphi_i^j(t)$ $(i = 1, 2, \ldots N; j = 1, 2, \ldots M)$ is the instantaneous phase of the system; M is the number of the oscillators in the common circle, same as the number of subgroups. In our study we fix the size of the common ring and the coupling radius as $M = 50$ oscillators $R_M = 20$, respectively.

Our choice of dynamical system is motivated by the fact that the nonlocal interaction in a network of KS phase oscillators is a paradigm of chimera states. The phase dynamics of each node φ_i^j (i^{th} node in the j^{th} subnetwork) in the network is described by the following

$$\frac{d\varphi_i^j}{dt} = \begin{cases} \omega_i^j - \dfrac{\lambda}{1+2R^N} \displaystyle\sum_{k=i-R^N}^{i+R^N} \sin(\varphi_i^j - \varphi_k^j + \alpha), & i \neq 1, \\ \omega_i^j - \dfrac{\lambda}{1+2R^N} \displaystyle\sum_{k=i-R^N}^{i+R^N} \sin(\varphi_i^j - \varphi_k^j + \alpha) - \dfrac{\lambda}{1+2R^M} \displaystyle\sum_{k=i-R^M}^{i+R^M} \sin(\varphi_i^j - \varphi_i^k + \alpha), & i = 1. \end{cases} \tag{1}$$

where ω_i^j is the natural frequency of the i^{th} oscillator in j^{th} subnetwork, λ define the coupling strength, which is the same in all subnetworks and the global ring. respectively, R^N and R^M is defined as the number of neighboring oscillators each oscillator is connected to, in both direction, in each subnetwork and global ring, respectively; α is a phase lag parameter identical for all the oscillators. We consider $\omega_i^j = 1.0$ for all oscillators in the network.

While the subnetworks consist of only one node ($N = 1$) the system represents the classical non-locally coupled ring and demonstrate chimera states when the value of α remains close to $\pi/2$ for an appropriate choice of λ. The initial phase for oscillators in the global ring are the following:

$$\varphi_1^j(0) = \begin{cases} \pi\left(\dfrac{4i}{N} - 1\right), & i \in \left[0, \frac{N}{2}\right], \\ \pi\left(3 - \dfrac{4i}{N}\right), & i \in \left[\frac{N}{2} + 1, N\right]. \end{cases} \tag{2}$$

At the same time, then $N > 1$, the initial phase of oscillators in each subnetwork is identical:

$$\varphi_1^j(0) = \varphi_i^j(0), \forall i. \tag{3}$$

We also added small random fluctuations in the initial conditions. In order to confirm the existence of chimera states, we use a statistical measure, a strength of incoherence SI[?] from a local standard deviation analysis. The SI is defined as

$$SI = \frac{\sum_{r=1}^{m} \Theta(\delta - \sigma_r)}{m}, \tag{4}$$

where $\Theta(\bullet)$ is the Heaviside step function, δ is a predefined threshold value, m is the number of oscillators in each group of equal length $n = N/m$, for which the local standard derivation σ_r is calculated as

$$\sigma_r = \left\langle \sqrt{\frac{1}{n} \sum_{s=n(r-1)+1}^{rn} (\varphi_1^j - \Phi)^2} \right\rangle_t. \tag{5}$$

Here $\langle\bullet\rangle_t$ denotes averaging over time and Φ corresponds to the phase averaged over all the oscillators in the global ring. A value of $SI^j = 0$ in each layer represents a coherent state while $SI = 1$ and $0 < SI < 1$ represents incoherent and chimera states respectively. To reveal, how the existence of chimera state correlates with the dymanical regimes in different scales of the network, we calculate the classical order parameter within the common circle of oscillators:

$$r^M = \langle \frac{1}{M} \left| \sum_{j=1}^{M} e^{i\phi_1^j(t)} \right| \rangle_t, \tag{6}$$

and the order parameter averaged over subnetworks

$$r^N = \langle \frac{1}{MN} \sum_{j=1}^{M} \left| \sum_{i=1}^{N} e^{i\phi_i^j(t)} \right| \rangle_t. \tag{7}$$

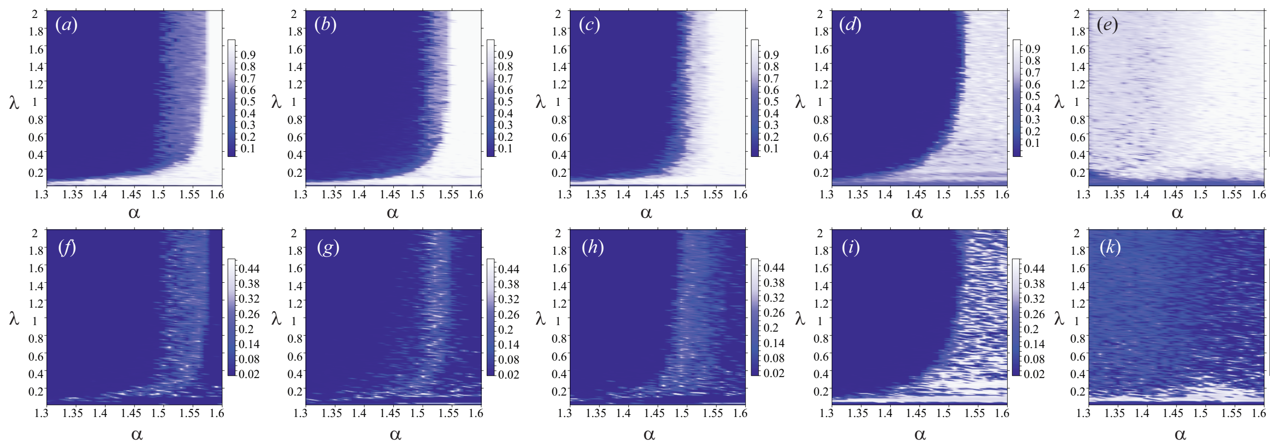

Figure 2. (Color online) The strength of incoherence (SI) (a-e) and the discontinuity measure (DM) (f-k) versus phase lag (α) and coupling strength (λ) for various configurations of the network: (a, f) $N = 1$; (b, g) $N = 5$, $R_N = 1$; (c, h) $N = 5$, $R_N = 2$; (d, i) $N = 20$, $R_N = 1$; (e, k) $N = 20$, $R_N = 5$.

3. RESULTS

The Fig. 2 shows the phase diagram measured using the strength of incoherence (SI) value (a-e) in parameter plane (α,λ) and the discontinuity measure (DM) for different sizes and topologies of the subgroups of nodes. When $N = 1$ (a), we can see using the values of SI and DM, that chimera emerge at $\lambda = 0.02$ and exists in the region $\alpha \in [1.5; 1.55]$. The further growth of the coupling strength, λ, does not introduce any changes in dynamical regime. This figure shows that the chimera state is characterized by SI values within $[0,5; 0,6]$. In Fig. 3 we show the order parameter for the common circle (a-e) and for the subgroups (f-i) (see Eq. 6). Note, that there is no value of subgroup order parameter when $N = 1$. Fig. 3 (a) reveals that chimera state emerge than common circle order parameter, takes values near ≈ 0.7.

Next we consider four systems,(b, f) $N = 5$, $R_N = 1$; (c, g) $N = 5$, $R_N = 2$; (d, h) $N = 20$, $R_N = 1$; (e, i) $N = 20$, $R_N = 5$, i.e. each oscillator from common circle is also located in its subgroup of locally coupled oscillators, and examine how such topology will effect the chimera region. In this case, (b, f) $N = 5$, $R_N = 1$, the region of chimera state becomes thinner than in the considered $N = 1$ case and characterized by a lower order parameter in the common circle ≈ 0.6. This time the order parameter in the subgroups is equal to 0.8. In the following case, (c, g) $N = 5$, $R_N = 2$, we increase the number of neighboring oscillators in the subgroups and see that the order parameter in the commpn ring has decreased, in comparison with the case above. This phenomenon can be considered as evidence of competition-like interaction between the common ring and locally coupled groups.

In the last two cases, a sharp increase in the size of subgroups led to the following results. When $N = 20$, $R_N = 1$, the chimera state region narrowed very strongly. Here we begin to see a clear divinding line from a coherent to an incoherent state in the order parameter diagrams, Fig. 3 (d, h), as well as in SI values Fig. 2 (d). Next we increase the coupling radius in the subgroups to $R_N = 5$ (Fig. 3 (e, i)) and it leads to the appearance of an incoherent regime over the entire range of the parameter plane. The order parameter in the common ring fell to appreciably low values, while in the subgroups it slighly decreased in comparison with the last case, but the configuration of the parameter plain almost not changed. Thus, we can say that increasing the size of subgroups and the radius of non-local communication within them leads to increasing of the competition between them and the common network's ring. This results to a decrease and the subsequent disappearance of the region of the chimera state emergence.

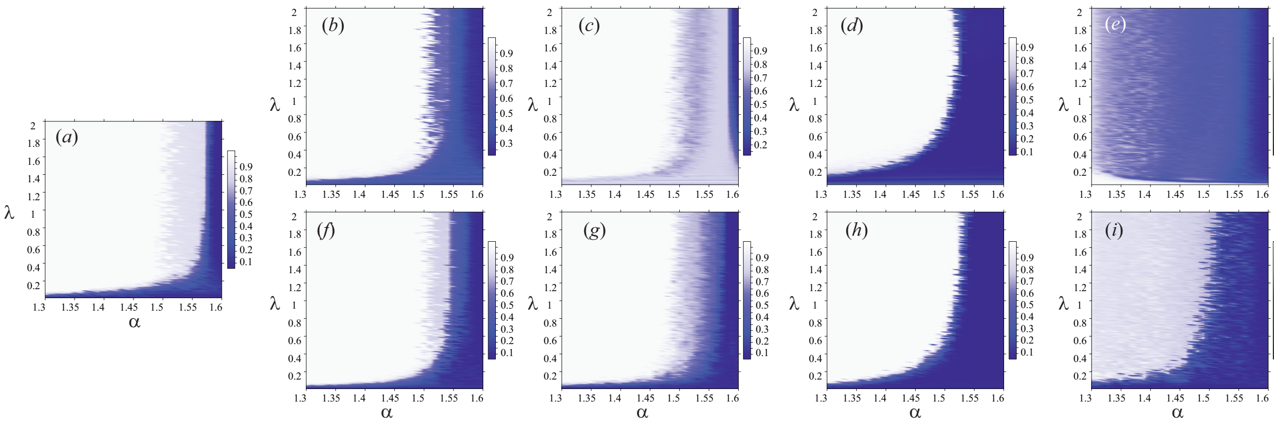

Figure 3. (Color online) The order parameter in the common circle (a-e) and in the subgroups (f-i) versus phase lag (α) and coupling strength (λ) for various configurations of the network: (a) $N = 1$; (b, f) $N = 5$, $R_N = 1$; (c, g) $N = 5$, $R_N = 2$; (d, h) $N = 20$, $R_N = 1$; (e, i) $N = 20$, $R_N = 5$

4. CONCLUSION

We have numerically studied the dynamical regimes which arise in the multi-scale network of nonlocally coupled oscillators. In particular, we examine how the emergence of the additional local scale-interaction effects the appearance of chimera states. We show that the increase of nodes in subgroups leads to the destruction of the synchronous interaction within the common ring and to the narrowing of the chimera region. At the same time, the observed effects are realizing in the condition of the same coupling in the subgroups of oscillators and the common ring. One can suppose, that ingomogenious coupling between the different scales of real systems can produce the inverse effect. We think, that described phenomena could take place in real-world networks, which are consist of interacting structures.[9, 20–24, 29–33]

5. ACKNOWLEDGMENTS

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