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Transition to Chaos and Chaotic Generation in a Semiconductor Superlattice Coupled to an External Resonator

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Abstract—The transition to chaos in a semiconductor superlattice coupled to an external resonator is studied. It is shown that the transition to chaos proceeds through intermittency. It is found that the system exhibits broadband generation regimes, which is of direct interest for the practical use of nanostructures in data transmission systems.

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INTRODUCTION

Semiconductor superlattices are nanostructures consisting of several (and often several tens or more) alternating semiconductor materials with different widths of the forbidden band. They were first proposed
1 by L. Esaki and R. Tsu [1, 2] and independently in [3] as one-dimensional structures for studying different quantum effects associated with resonance tunneling and Bloch fluctuations. After the publication of these original works, different types of semiconductor superlattices with different electromagnetic properties were proposed and experimentally executed. Semiconductor superlattices are now suitable objects for studying and understanding the processes of solid state physics [2, 4], and for studying different nonlinear phenomena [5–9]. In addition, Bloch fluctuations and domain transport in strongly coupled superlattices, and the nonlinear processes associated with them [10], make superlattices promising elements for the generation, enhancement, and detection of high-frequency (up to several tens of terahertz) signals [11].

In the context of using semiconductor superlattices in high-frequency electronics, studies of the interaction between a superlattice and the external electrodynamic systems with which the nanostructure can be coupled are of great importance. This statement of the problem can be considered in two aspects. First, it is impossible at high frequencies to eliminate parasitic capacities and inductances in the connection elements of a superlattice (wires, contacts, and so on) responsible for the parasitic resonance contours acting on the superlattice, so the effect of such external parasitic resonance contours must be taken into account when studying the generation modes of the superlattice. Second, it is well known that external electrodynamic systems are often an effective way of controlling complex nonlinear oscillatory processes in the super high frequency range. The introduction of additional resonance systems in particular can lead to the excitation of chaotic fluctuations in generators (e.g., in resonance backward-wave tubes [12]).

Earlier, we showed that connecting an external quality resonance system stimulates the emerges of chaotic and quasiperiodic modes of fluctuation in a semiconductor superlattice [13–15]; this is of great interest in developing systems for the covert transmission of information and super-fast random number generators. Studies of the mechanism behind the transition to chaos in a given system are of interest from the viewpoint of both nonlinear dynamics and application in systems for the covert transmission of information.

In this work, we study in detail the transition to chaos in a semiconductor superlattice coupled to an external resonator. It is shown that the transition to chaos occurs according to an intermittency scenario. It is also found that the system exhibits chaotic broadband generation; this is of direct interest when using a superlattice as a generator in data transmission systems.

SYSTEM AND NUMERICAL MODEL

To describe the collective dynamics of a charge in a semiconductor superlattice, we use a standard model based on a self-consistent Poisson system and continuity equations integrated numerically. The parame-

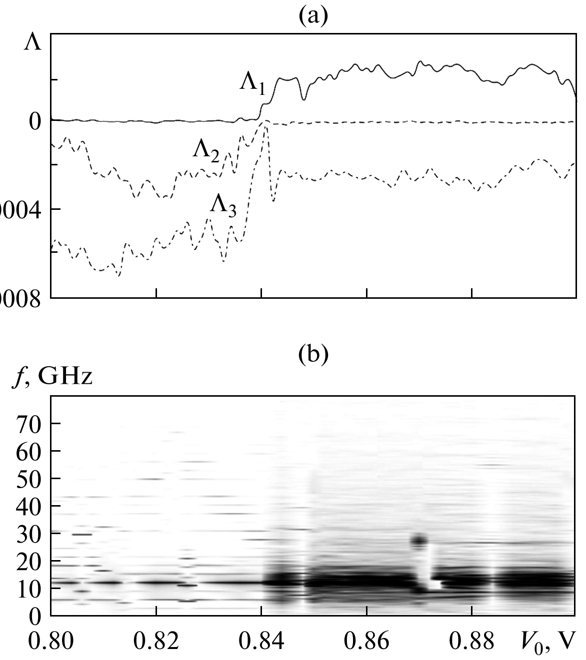

Fig. 1. (a) Dependences of three senior Lyapunov exponents on the voltage of the superlattice power supply. (b) Spectral composition of the voltage fluctuations in the resonator upon changes in the voltage of the power supply. Frequency of external resonator $f_Q = 13.81$ GHz; quality $Q = 150$.

ters of the analyzed superlattice are analogous to the superlattices described in [6, 9], where it was assumed that the conductive region of the minizone was divided into $N = 480$ layers whose widths were quite narrow: $\Delta x = L/N = 0.24$ nm.

The variation in the charge density of each layer (the right-hand boundary of which is $x = m\Delta x$) is given by the discrete analog of the equation of current continuity,

$$e\Delta x\frac{dn_m}{dt} = J_{m-1} - J_m, m = 1, \ldots N, \quad (1)$$

where e is the electron charge, J_{m-1} and J_m are the current density on the left- and right-hand boundaries of layer m. The current density was determined as

$$J_m = en_m \upsilon_d(\bar{F}_m), \quad (2)$$

where $\bar{F}_m$ is the average electric field in layer m. Drift velocity $\upsilon_d(\bar{F}_m)$ was determined from the relation

$$\upsilon_d = \frac{d\Delta}{2\hbar}\frac{\tau\omega_B}{(1+\tau^2\omega_B^2)}, \quad (3)$$

where $\hbar$ is the Planck constant, τ is the speed of electron scattering, and $\omega_B = eFd/\hbar$ is the angular frequency of the Bloch fluctuations of electrons [1, 8].

Electric field F_m at the boundary of layer m can be determined using the Poisson equation in discrete form

$$F_{m-1} = \frac{e\Delta x}{\varepsilon_0\varepsilon_r}(n_m - n_D) + F_m, m = 1, \ldots N, \quad (4)$$

where $n_D = 3 \times 10^{22}$ m^{-3} is the density of doping in the layers of the superlattice.

Ohmic boundary conditions were used to determine current $J_0 = \sigma F_0$ in a highly doped emitter with electric conductivity $\sigma = 3788\ \Omega^{-1}$. Voltage V_{sl} applied to the device was determined from the expression

$$V_{sl} = U + \frac{\Delta x}{2}\sum_{m=1}^{N}(F_m + F_{m+1}), \quad (5)$$

where U is the drop in voltage on contacts with allowance for the formation of layers with increased charge concentration near the emitter and the lowered charge concentration near the collector of the superlattice [6]. Knowing the current density in each layer, we can calculate the total current passing through the superlattice [10]:

$$I(t) = \frac{A}{N+1}\sum_{m=0}^{N} J_m, \quad (6)$$

where $A = 5 \times 10^{-22}$ m^2 is the cross section of the superlattice. Note that in numerical modeling, we assume that the superlattice is at a temperature low enough for the diffusion component of current density to be ignored.

We use the single-mode approximation to simulate the external resonance contour. The resonator is described by an equivalent scheme for which the Kirchhoff equations take the form

$$C\frac{dV_1}{dt} = I(V_{sl}) - I_1, \quad (7)$$

$$L\frac{dI_1}{dt} = V_{sl} - V_0 - R_1 I_1 + I(V_{sl})R_l, \quad (8)$$

where $I(V_{sl})$ is the current generated by the superlattice. The resonator is characterized by frequency f_Q and quality Q.

SYSTEM DYNAMICS

Three senior Lyapunov exponents were calculated to study the nonlinear dynamics of the system. Their dependences on the voltage of the power supply are shown in Fig. 1a. It is clear that the periodic mode is observed to 840 mV: the first Lyapunov exponent is zero, and the second and the third factors are negative. When the voltage is raised 840 mV, the values of the second and the third factors begin to grow sharply and reach zero in the bifurcation point. The first factor begins to assume positive values. Upon a further

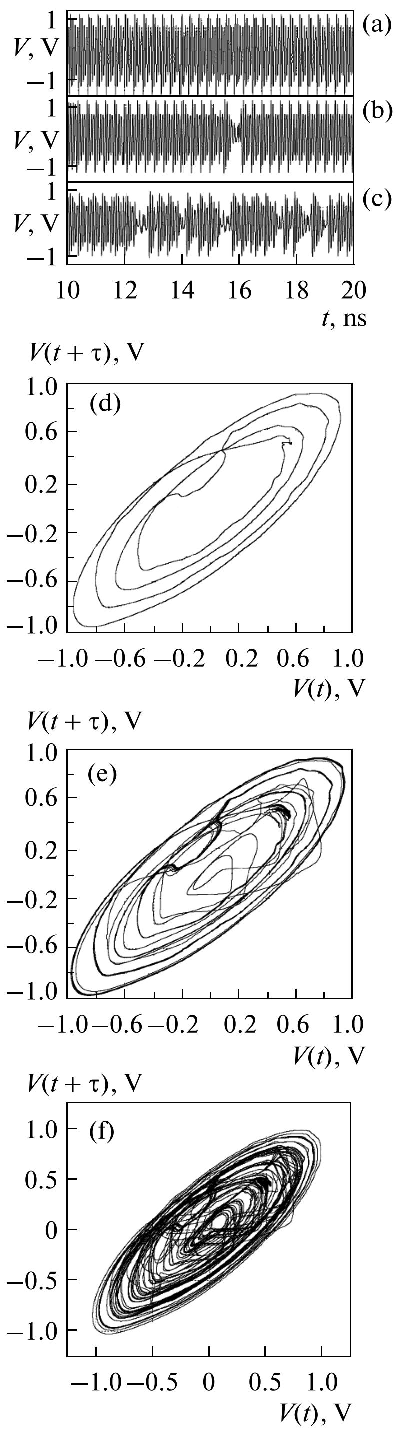

Fig. 2. (a–c) Time implementations and (d–f) phase portraits of fluctuations in voltage in the resonator upon a rise in power supply voltage: (a, d) V_0 = 844 mV; (b, e) V_0 = 846.6 mV; (c, f) V_0 = 848 mV. Frequency of external resonator f_Q = 13.81 GHz; quality Q = 150.

increase in voltage, the value of the third Lyapunov exponent becomes negative. This behavior of the Lyapunov exponents indicates the onset of chaotic dynamics at voltages above 840 mV.

To illustrate this mode, Fig. 1b shows the variation in the spectrum of voltage fluctuations in the system as a function of the voltage of the power supply. Before the transition to chaos, only the main frequency and individual harmonics are observed in the spectrum. The spectrum broadens drastically at the bifurcation point, and the main harmonic becomes very noisy. This mode is of obvious interest for using the semiconductor superlattice as a generator of broadband signals in an external resonance system.

Let us consider the change in the dynamics of the system in the region of the transition to chaos. Figure 2 shows the time implementations (a–c) and phase portraits (d–f) of the voltage fluctuations on the superlattice in the resonator when the voltage of the system power supply increases. It can be seen that a complicated periodic mode emerges in the system at a voltage of 844 mV: only regular fluctuations are observed in implementation (a), and phase portrait (d) is the limiting cycle of period four.

A short turbulent phase appears in time implementation (b) when the voltage rises (V_0 = 846.6 mV); it is seen clearly in phase portrait (d). The number of turbulent phases in time implementation (d) grows sharply upon a further increase in the voltage of the power supply (V_0 = 848 mV), and phase portrait (f) starts to acquire the form of a chaotic attractor. We emphasize that this mode is characterized by a chaotic broadband radiation spectrum (Fig. 1b); this is of great interest for application in data transmission systems.

CONCLUSIONS

The chaotic dynamics and transition to chaos in a semiconductor superlattice coupled to an external quality resonator were studied. Three senior Lyapunov exponents were calculated as a function of the voltage of the semiconductor superlattice's power supply. It was shown that the transition to chaotic dynamics occurs according to an intermittency scenario. In addition, this intermittent behavior results in substantial broadening of the spectrum, which is of interest both for the fundamental study of semiconductor nanostructures and for the practical application of superlattices in generating microwave and terahertz fluctuations and their use in systems for the transmission of covert information.

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