Route to Coherence in a Frequency-Heterogeneous Kuramoto Network

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***Abstract*—Synchronization phenomena in populations of interacting elements are the subject of extensive research in biological, chemical, physical and social systems. Understanding of the emergent behavior in controlled experiments or real systems requires a focus on the consideration of heterogeneous network models. In this study, we explore the influence of the network topology on the rout to coherence in a heterogeneous Kuramoto model under the monotonically increasing coupling strength. Specifically, we consider regular (non-locally coupled ring), small-world (SW) and scale-free (SF) networks. We demonstrate a specific type of a chimera-like behavior that emerges in all three types of topology under certain conditions.**

Keywords—complex networks, synchronization, chimera, heterogeneity, Kuramoto model

I. Introduction

Synchronization phenomena in populations of interacting elements are the subject of extensive research in biological, chemical, physical and social systems [1]. The process of synchronization refers to the adjustment of rhythms of interacting oscillatory systems, whereas chimera states are characterized by the fascinating coexistence of coherent and incoherent sub-populations in networks of coupled oscillators [2-11].

On another note, discontinuous or explosive transitions to coherence in networks are receiving growing attention these days [12-16]. The paradigmatic Kuramoto model being able to provide the most effective approach to explain how synchronous behavior emerges in complex systems, there exists significant attempts in exploring both chimera states and explosive transition to synchrony. But, in most of the studies, these two processes have been studied exclusively, without paying attention to a possibility in linking them.

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In contrast to approaches solely concentrating on abrupt transitions to synchrony and the associated hysteresis, we here put forward the emergence of chimera-like behavior on the route to an explosive transition in networks of coupled Kuramoto phase oscillators. Complex systems naturally display heterogeneity in its constituents, so in this paper, we explore route to coherence in a heterogeneous Kuramoto model with different topology of the inter-element coupling. We reveal a specific type of a partially coherent behavior that is excited under weakly non-local, small-world and sparse scale-free coupling and suppressed in globally coupled, strongly rewired and dense scale-free networks.

II. Numerical Model

We consider a network of N = 100 phase oscillators, in which the dynamics of each node is represented by the following form of the Kuramoto equation:

$$\dot{\phi}_i = \omega_i + \lambda R_i \sum_{l=1}^{N} A_{il} sin(\phi_l - \phi_i), \quad (1)$$

$$R_i = \frac{1}{k_i} |\sum_{l=1}^{N} A_{il} e^{j\phi_l}|, \quad (2)$$

where ϕ_i, ω_i and k_i are the phase, natural frequency and the degree of the i^{th} Kuramoto oscillator respectively, also $j = \sqrt{-1}$. The matrix $A = [A_{il}]$ is the underlying graph adjacency, generated using either Watts-Strogatz (WS) algorithm [17] in the case of the regular and small-world (SW) coupling or Barabasi-Albert (BA) algorithm [18] in the case of scale-free (SF) network. The parameter λ is the overall coupling strength. R_i represents the local order parameter and evaluates the degree of coherence in the neighborhood of the i^{th} element. It contributes adiabatically to the coupling term and provides the mechanism for explosive synchronization. The values of ω_i are uniformly distributed from 9 to 11.

To quantify the network's coherence, we use the averaged global order parameter as:

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$$R = \frac{1}{N(t_{max}-t_{trans})}\int_{t_{trans}}^{t_{max}} \left|\sum_{l=1}^{N} e^{j\phi_l(t)}\right| dt, \quad (3)$$

where $t_{trans} = 1500$ and $t_{max} = 2000$ respectively denote the transient time and maximal simulation time. Moreover, we illustrate the collective behavior of the Kuramoto model using the mean effective frequency $\langle\dot{\phi}_i\rangle$ defined by time averaging instantaneous effective frequency $\dot{\phi}_i(t)$ after the transient process. The network model simulation is conducted using the Runge-Kutta method of order 5(4) [19] implemented in the Differential Equation Solver for Julia programming language [20]. To control the accuracy of the numerical integration, we use the adaptive time-stepping with relative tolerance parameter equal to 10^{-6}.

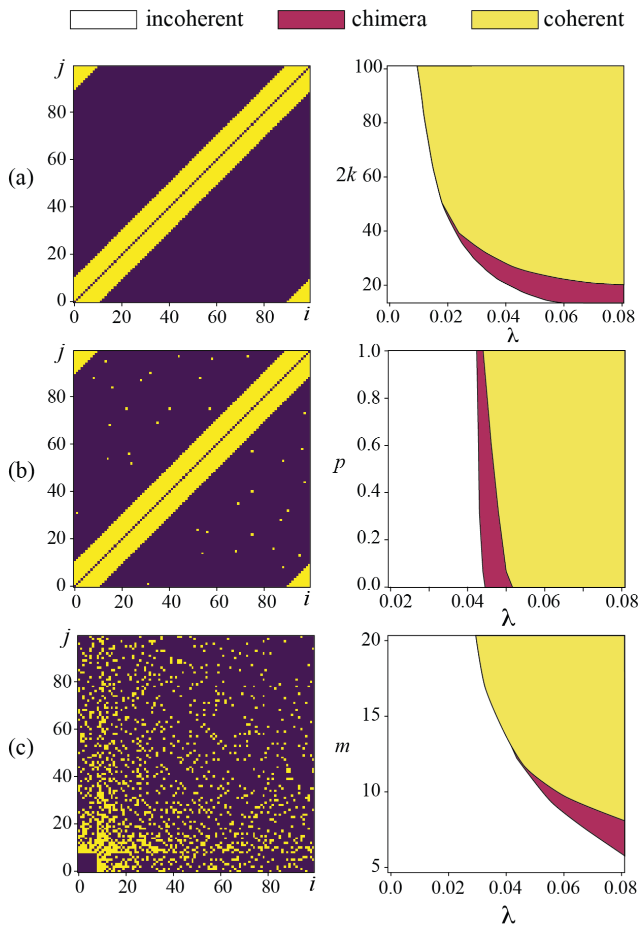

Fig. 1. Adjacency matrix (left column) and the two-parameter phase diagram (right column) illustrating the route to coherence for different types of network topology: (a) regular non-local coupling; (b) small-world (SW); (c) scale-free (SF). Phase diagrams are color coded by the macroscopic network state.

III. Results

First, we explore how the number of the nearest neighbors k in the WS model affects the route to coherence in the regular non-locally coupled Kuramoto network (Fig. 1a). The increase of the nearest neighbors k ($2k \geq 40$) suppresses the emergence of the chimera state. Thus, the strong interaction within a large group of elements leads to the explosive transition directly from the incoherent to a globally frequency-locked state in the absence of the intermediate partially-coherent state. On the contrary, the decrease of k promotes weaker interaction between network elements and makes it of a more local kind. These factors strengthen the influence of the network's heterogeneity, slow down the transition to coherence and support the partial coherence in a wider range of λ.

We also consider how the structural properties of the SW and SF graphs affects the transitions of the collective behaviors. It is seen in Fig. 1b, that in the case of SW topology, the increase of rewiring probability p lowers the critical value of the coupling strength providing the explosive transition and smooths the area of partially coherent state. In the limit case of $p = 1.0$ (completely random rewiring), the intermediate partially coherent state is suppressed by the increased network randomness resulting in the direct explosive transition from the incoherent dynamics to a frequency-locked (π-state) at $\lambda = 0.0405$. Accordingly, in the SF network, the partially coherent behavior is only possible in sparsely connected graphs ($m < 12$) (Fig. 4c). For the dense coupling $m \geq 12$, only an explosive transition is observed.

Taken together, these results demonstrate that the detected partially coherent behavior in a heterogeneous Kuramoto model could be suppressed (i) by the increase of the neighborhood in the case of non-local coupling, (ii) by a strong rewiring in the SW network and (iii) by growing a densely coupled SF graph. We argue that these ways share a similar mechanism based on the establishment of the long-scale coupling between the network elements. Thus, the effect of initial heterogeneity of network oscillators could be annihilated by expanding the coupling area for each element, that provides the dominance of the attractive mechanisms [21].

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