Chaos and hyperchaos in a chain of coupled Rydberg atoms

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***Abstract*—Partially quantum coherent arrays of artificial atoms (e.g., superconducting qubits) may be used as analogue simulators of other quantum systems, when the behaviour of the latter cannot be directly observed or modeled by classical means. Here we show that a chain of superconducting qubits can simulate the chaotic behaviour predicted in chains of Rydberg atoms. We investigate the transition from periodic to chaotic dynamics and show the increase of the number of positive Lyapunov exponents as the number of atoms grows. The possibility of an external control and suppression of hyperchaos in the chain is demonstrated.**

Keywords—Rydberg atoms, chaos, hyperchaos, qubits, nonlinear dynamics..

I. Introduction

Current progress in fabrication and experimental techniques allows creation of large, partially quantum coherent arrays of ``artificial atoms" (e.g., superconducting qubits), which allow a degree of control over their quantum state. When fully developed, such devices can find numerous applications, ranging from quantum sensing and communications to universal quantum computing [1,2]. A straightforward application of such structures is the simulation of other quantum systems, which are too big or too complex for simulation by classical computers. The digital quantum simulation is based on mimicking the action of the evolution operator of the simulated system with a sequence of elementary transformations realized by timely changing the Hamiltonian of the simulator. While providing scalability, versatility and controlled accuracy, this approach imposes practically as strict requirements on the hardware as a universal quantum computation. On the contrary, the analogue quantum simulation is less demanding, because it uses the natural evolution of the simulator and thus does not require a precise control over its Hamiltonian. The downside is the loss of universality and controlled precision. This approach has already provided interesting results within the current experimental capabilities [3].

In this paper we discuss the possibility of simulating a chain of Rydberg atoms with qubits. Rydberg atoms are a topic of active research in quantum technologies and many-body physics because of their strong dipole-dipole interactions allowing long-range entanglement, leading to a rich physics, and also, as we shall see, they are a natural choice for analogue quantum simulation.

II. Model

The standard description of a set of interacting qubits is based on the Ising Hamiltonian,

$$H = \sum_j H_j + \sum_{j<k} U_{jk}, \tag{1}$$

with

$$H_j = -\tfrac{1}{2}\left[\epsilon_j \sigma_j^z + \delta_j \sigma_j^x\right], \quad U_{jk} = J_{jk}\sigma_j^z \sigma_k^z. \tag{2}$$

Here the Pauli operators, σ_j^α, describe the transitions between qubit states, ϵ_j and δ_j are, respectively, the bias and the tunneling matrix element in the j-th qubit, and J_{jk} is the qubit-qubit coupling. Expressing the Pauli matrices in terms of projectors to ground and excited states of the qubits, $|g\rangle$, $|e\rangle$,

$$\sigma_j^z \equiv |g\rangle\langle g|_j - |e\rangle\langle e|_j \equiv \hat{1}_j - 2|e\rangle\langle e|_j;$$
$$\sigma_j^x \equiv |e\rangle\langle g|_j + |g\rangle\langle e|_j; \hat{1}_j \equiv |g\rangle\langle g|_j + |e\rangle\langle e|_j \tag{3}$$

we can rewrite (2) as

$$H_j = -\tfrac{1}{2}\epsilon_j \hat{1}_j + \epsilon_j |e\rangle\langle e|_j - \tfrac{1}{2}\delta_j\left[|e\rangle\langle g|_j + |g\rangle\langle e|_j\right],$$
$$H_{jk} = \left[\hat{1}_j - 2|e\rangle\langle e|_j\right] \otimes \left[\hat{1}_k - 2|e\rangle\langle e|_k\right]. \tag{4}$$

The resulting form of the Hamiltonian (1),

$$H = \sum_j \left\{\left[\epsilon_j - 2\sum_k J_{jk}\right]|e\rangle\langle e|_j - \tfrac{1}{2}\delta_j\left[|e\rangle\langle g|_j + |g\rangle\langle e|_j\right]\right\}$$
$$+4\sum_{jk} J_{jk}\, |e\rangle\langle e|_j \otimes |e\rangle\langle e|_k + \left[-\tfrac{1}{2}\sum_j \epsilon_j + \sum_{jk} J_{jk}\right].$$

Up to the irrelevant total energy shift, $\Delta E = \left[-\tfrac{1}{2}\sum_j \epsilon_j + \sum_{jk} J_{jk}\right]$, and parameter relabeling, this Hamiltonian is just an inhomogeneous version of the interaction-representation Hamiltonian of a system of Rydberg atoms in a laser field [4,5].

$$H = \sum_{j=1}^{N}\left[-\tilde{\Delta}|e\rangle\langle e|_j + \tfrac{\tilde{\Omega}}{2}\left(|e\rangle\langle g|_j + |g\rangle\langle e|_j\right)\right]$$
$$+\tfrac{V}{N-1}\sum_{j<k}|e\rangle\langle e|_j \otimes |e\rangle\langle e|_k \tag{5}$$

where $\tilde{\Delta} = w_l - w_o$ is the detuning between the laser and transition frequencies, and $\tilde{\Omega}$ is the Rabi frequency (tuned by the laser field amplitude). Therefore, one can directly model the behavior of Rydberg atoms by using a set of qubits, thus realizing an instance of analogue quantum simulation.

Further, introducing appropriate Lindblad operators responsible for relaxation and pure dephasing, we obtain for the matrix elements $w_j = (\rho_j)_{11} - (\rho_j)_{00}$ (population inversion of the j-th qubit) and $q_j = (\rho_j)_{10} = (\rho_j)^*_{01}$ (its coherence) the following set of equations

$$\dot{w}_j = -2\Omega\,\mathrm{Im} q_j - (w_j + 1)$$
$$\dot{q}_j = i\left[\Delta - c\sum_{k\neq j}(w_j + 1)\right]q_j - \tfrac{1}{2}q_j + i\tfrac{\Omega}{2}w_j \tag{6}$$

where w_j is the inversion, q_j is off-diagonal element, $\Omega = \tilde{\Omega}/\gamma$, $\Delta = \tilde{\Delta}/\gamma$, $c = dV/\gamma$, where d is the lattice dimension.

III. Results

We study the dynamics of ring lattices consisting of different number of Rydberg atoms as it is schematically shown in Fig. 1. We investigate the dynamics of the system using Lyapunov exponents calculation and bifurcation diagrams plotting. Authors of [4] found that there are 3 possible regimes of the dynamics: uniform, antiferromagnetic and oscillatory and no chaos. But we found chaos and hyperchaos in the system, and hyperchaos characterized by different number of positive Lyapunov exponents that depends of control parameters value and number of atoms in the system [6]. We analyse the region of control parameters Ω and Δ using Lyapunov exponents calculation.

We started our consideration with studying the dynamics of two coupled atoms. Previously, it has been shown [4] that interplay between the energy pumping and dissipation can eventually evoke self-sustained oscillations of the populations in this system, and even lead to emergence of bistability, when homogeneous and antiferromagnetic states coexist at the same time. Our analysis revealed another interesting phenomenon associated with onset of deterministic chaos, which is realized via a cascade of period-doubling bifurcation for periodic oscillations. We found out that the discovered chaos is a robust phenomenon existing in ranges of parameters, which for certain parameter values can coexist with the antiferromagnetic steady state [7].

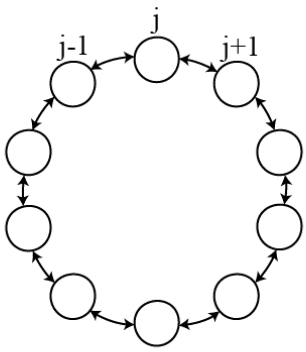

Fig. 1. The model of circular chain of 10 Rydberg atoms.

Analysis of the chains with even larger *N* revealed that hyperchaos is not only preserved in the system, but becomes more complicated as more Lyapunov exponents become positive. The result of our analysis in summarized in Fig. 2, where the dependence of the number of positive Lyapunov exponents *M* on the number of the atoms in the chain is depicted. The graphs demonstrates almost linear growth when inclusion of two-three additional atoms leads to appearance of one more positive Lyapunov exponent. This phenomenon takes place due to almost no correlation between the oscillations in distant atoms, which creates the condition, when an inclusion of a subsystem of coupled atoms, able to demonstrate chaotic dynamics, simply adds one more positive value in the spectrum of the Lyapunov exponents.

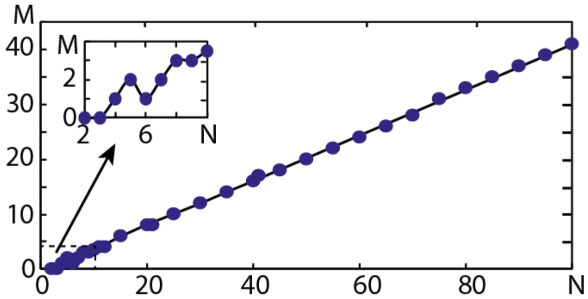

Fig. 2. The dependance of maximum number of positive Lyapunov exponents for Rydberg atom chains from chain length.

For the chain of 5 atoms we also plotted a regime map. It was found that there are a lot of different regimes such as hyperchaos, chaos, quasiperiodic, periodic and stationary regimes [8]. The area of parameters Ω and Δ of oscillatory dynamics for chain of 5 atoms are bigger that one for 2 atoms. Investigating the transition to chaos in the system we calculated bifurcation diagram. As in system of 2 atoms there is a transition from periodic regime to chaos occurs through a direct cascade of period doubling bifurcations. But the first one is arising from a periodic window of quasiperiodic regime.

IV. Conclution

In conclusion we discovered that an interplay between dissipation and energy pumping in the quantum coherent systems can evoke very non-trivial emergent phenomena associated with onset of complex stable chaotic and even hyperchaotic oscillations. The complexity of the hyperchaos increases with the size of the chain.

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