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Coexistence of Two Different Types of Intermittency near the Boundary of Phase Synchronization in the Presence of Noise

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***Abstract*—**Intermittent behavior near the boundary of phase synchronization in the presence of noise is studied. In certain range of the coupling parameter and noise intensity the intermittency of eyelet and ring intermittencies is shown to take place. Main results are illustrated using the example of two unidirectional coupled Rössler systems. Similar behavior is shown to take place in two hydrodynamical models of Pierce diode coupled unidirectional.

***Keywords*—**Chaotic oscillators, phase synchronization, noise, intermittency of intermittencies, control.

I. Introduction

INTERMITTENCY is an ubiquitous phenomenon in nonlinear science [1]. It is observed in different systems including the physical, physiological and biological ones (see, e.g., [2]-[8]). Several types of the intermittent behavior are traditionally distinguished, among which there are type I-III [1], [9], on-off [10], eyelet [5], [11] and ring [12] intermittencies. Despite of the different mechanisms resulting in the onset of the types of intermittency mentioned above and their different statistical characteristics in all known cases for the fixed values of the control parameters time series contains only two different type of the behavior alternating with each other. Later it has been shown that for several conditions, for example, near the boundary of phase synchronization of non-autonomous or coupled chaotic oscillators and systems demonstrating periodic dynamics in the presence of noise in the certain range of the time scales of observation the coexistence of two different types of intermittent behavior is possible to exist. Such regime has been called as intermittency of intermittencies [13]. It is a principally new level of complexity in the nonlinear system dynamics and therefore is worth of investigation.

In [13] the general theory of coexistence of two different types of intermittent behavior in nonlinear systems has been developed. For the case of intermittency of eyelet (type I-intermittency in the presence of noise in supercritical region of the control parameter) and ring intermittencies the analytical expressions for the laminar phase lengths for the fixed value of the control parameter and dependence of the mean length of laminar phases on the criticality parameter have been deduced. The results of numerical simulation of chaotic systems and oscillators demonstrating periodic dynamics in the presence of noise being near the boundary of phase synchronization in the certain range of time scales are in a good agreement with the results of theoretical predictions.

In the present Report we analyze the possibility of existence of intermittency of intermittencies in chaotic systems being near the boundary of phase synchronization in the presence of noise. As it would be shown below, in the certain range of the control parameter values and intensity of noise the coexistence of eyelet and ring intermittencies takes place.

II. Rössler Systems

Let us consider characteristics of intermittency of intermittencies near the boundary of phase synchronization in the presence of noise using the example of two unidirectional coupled Rössler systems. The system under study is given by

$$\begin{aligned}
\dot{x}_1 &= -\omega_1 y_1 - z_1,\\
\dot{y}_1 &= \omega_1 x_1 + a y_1,\\
\dot{z}_1 &= p + z_1(x_1 - c),\\
\dot{x}_2 &= -\omega_2 y_2 - z_2 + \varepsilon(x_1 - x_2),\\
\dot{y}_2 &= \omega_2 x_2 + a y_2 + D\xi,\\
\dot{z}_2 &= p + z_2(x_2 - c),
\end{aligned} \tag{1}$$

where $\mathbf{x}_{1,2}(t) = (x_{1,2}, y_{1,2}, z_{1,2})^T$ are state vectors of the drive and response systems, respectively, $a = 0.15$, $p = 0.2$, $c = 10$, $\omega_1=0.93$, $\omega_2=0.95$ are the control parameter values, $\xi(t)$ is a random Gaussian process with zero mean and unit variance, D is a noise intensity. To integrate the stochastic differential equations (1) we have used the four order Runge-Kutta method adapted for the stochastic differential equations [14] with time discretization step $\Delta t = 0.001$. To detect the phase synchronization regime we have analyzed the phase differences of interacting systems and testified the phase locking condition

$$|\phi(t)| = |\phi_1(t) - \phi_2(t)| < \text{const}. \tag{2}$$

The phases $\phi_{1,2}(t)$ of chaotic signals have been introduced into consideration as rotation angles on $(x_{1,2}(t), y_{1,2}(t))$-planes [15].

First of all we analyze the influence of the noise intensity on the boundary value of the phase synchronization regime

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onset in system (1). The results of our calculations show that if the noise intensity exceeds the certain critical value the synchronous regime starts destructing due to the loss of the phase coherence of the response system attractor. It is clear that in the fields where the boundary of the synchronous regime is not changed dramatically ($D \le 9$) the noise will not affect sufficiently both on the boundary of the synchronous regime onset and characteristics of intermittency taking place near that boundary. At the same time, in the field of the loss of the phase-coherence of the response system attractor ($D \ge 9$) the noise is able to bring new features in the characteristics of intermittency.

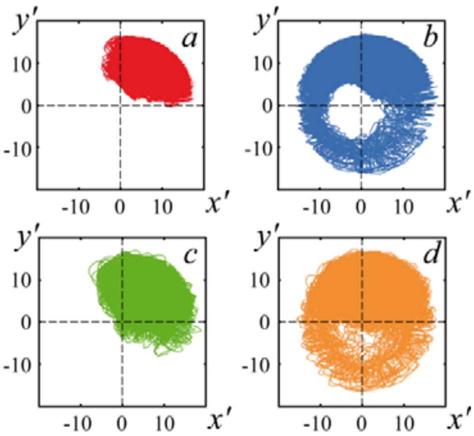

Fig. 1 Phase trajectories of the response Rössler system on the rotating plane (x', y'): (a) $\varepsilon = 0.045$, $D = 1.5$ - the phase synchronization regime, (b) $\varepsilon = 0.037$, $D = 1.5$ - eyelet intermittency, (c) $\varepsilon = 0.045$, $D = 10$ - ring intermittency, (d) $\varepsilon = 0.037$, $D = 10$ - intermittency of eyelet and ring intermittencies

To analyze the characteristics of intermittency taking place in the systems under study let us analyze the behavior of the response systems on the rotating plane in the same way as it has been done in [12], [16]:

$$\begin{aligned} x' &= x_2 \cos\phi_1 + y_2 \sin\phi_1, \\ y' &= -x_2 \sin\phi_1 + y_2 \cos\phi_1, \end{aligned} \tag{3}$$

where $\phi_1 = \phi_1(t)$ is the phase of the drive system, x_2, y_2 are the coordinates of the response system. In Fig. 1 phase trajectories of Rössler oscillators on the rotating plane (x',y') for different values of the noise intensity D and coupling parameter ε are shown. Fig. 1 (a) corresponds to the cases when in the system under study the synchronous dynamics takes place. In this case the response system attractor is phase-coherent and phase trajectory of the response system looks like a smeared fixed point which does not envelop origin of the rotating plane. Below the boundary of the phase synchronization in the case when the intensity of noise is small enough as in the case of the absence of noise the eyelet intermittency is observed (see Fig. 1 (b)). The response system attractor is also phase-coherent in this case and phase trajectory on the rotating plane is represented by the noised limit cycle. The increase of the noise intensity results in the loss of the phase coherence of the response system attractor that is accompanied by envelop of the origin in the rotating plane (Figs. 1 (c) and (d)). Envelop of the origin can be realized in two different ways. If the coupling parameter value exceeds the boundary value of the synchronous regime onset in the absence of noise but for the selected value of the noise intensity the phase synchronization does not exist the phase trajectory looks like a smeared fixed point enveloping origin (Fig. 1 (c)). In this case the ring intermittency takes place. If the coupling parameter is less than the boundary value of the phase synchronization in the absence of noise the phase trajectory on the rotating plane is represented by a smeared limit cycle enveloping origin (Fig. 1 (d)). In this case the coexistence of eyelet and ring intermittencies is observed.

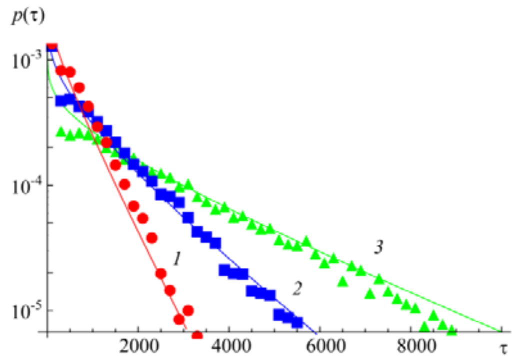

Fig. 2 Normalized distributions of the laminar phase length in the regime of intermittency of eyelet and ring intermittencies in two unidirectional coupled Rössler systems for different values of the control parameters (points) and their theoretical approximations (4) (solid lines): $1 - \varepsilon = 0.034$, $T_1 = 780$, $T_2 = 4500$; $2 - \varepsilon = 0.036$, $T_1 = 2700$, $T_2 = 4500$; $3 - \varepsilon = 0.038$, $T_1 = 12500$, $T_2 = 4500$

To confirm the presence of intermittency of intermittencies near the boundary of phase synchronization in the presence of noise let us analyze the statistical characteristics of intermittency, i.e. the distribution of the laminar phase lengths for the fixed values of the control parameters and dependence of the mean length of the laminar phases on the criticality parameter. In [13] we have shown that in the regime of coexistence of eyelet and ring intermittencies such distribution should obey the following relation:

$$\begin{aligned} p(\tau) = {} & \frac{\exp(-\tau/T_1)}{(T_1+T_2)}\left(1-\frac{\tau}{T_1}\right)\Gamma\left(0,\frac{\tau}{T_2}\right)+ \\ & +\frac{T_1^2+T_2^2}{T_1T_2(T_1+T_2)}\exp\left(-\frac{\tau}{T_1}-\frac{\tau}{T_2}\right)+ \\ & +\frac{\exp(-\tau/T_2)}{(T_1+T_2)}\left(1-\frac{\tau}{T_2}\right)\Gamma\left(0,\frac{\tau}{T_1}\right), \end{aligned} \tag{4}$$

where $\Gamma(a,z)$ is incomplete Γ-function. At that, the mean length of the laminar phases for this regime would be given by

$$T = -\frac{T_1^2 \log\left(\frac{T_1+T_2}{T_1}\right) - 2T_1T_2 + T_2^2 \log\left(\frac{T_1+T_2}{T_2}\right)}{T_1+T_2} \quad (5)$$

where $T_{1,2}$ can be obtained numerically for the regimes when the only one type of intermittent behavior (i.e. the eyelet or ring intermittencies) should exist [13].

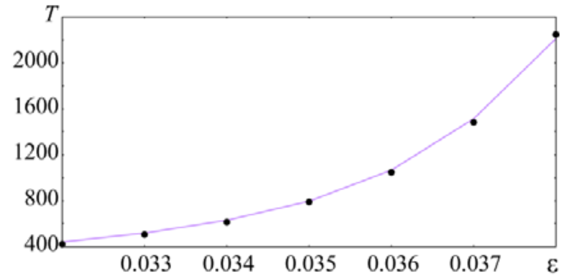

Fig. 3 Dependence of the mean length of the laminar phases on the coupling parameter in the regime of intermittency of eyelet and ring intermittencies ($D = 10$) and its theoretical approximation (5). The numerical data are marked by points; their theoretical approximation is shown by solid line

In Fig. 2 distributions of the laminar phase lengths in the regime of intermittency of intermittencies in the system under study for different values of the control parameters and their theoretical approximations are shown. It is clearly seen that in all considered cases the results of numerical simulations are in a good agreement with the results of theoretical studies that proofs the possibility of realization of the intermittency of intermittencies near the boundary of the phase synchronization regime in the presence of noise.

The additional confirmation of the presence of intermittency of eyelet and ring intermittencies near the boundary of the phase synchronization in the presence of noise is the dependence of the mean length of the laminar phases on the coupling parameter. Fig. 3 illustrates such dependence for the system under study and its theoretical approximation. It is clearly seen that the numerically obtained data are in a good agreement with the results of theoretical predictions.

III. Pierce Diodes

Similar results have been obtained for two unidirectional coupled hydro dynamical models of Pierce diode being in the presence of noise. The dynamics of such systems in the fluid electronic approximation is described by the self-congruent system of dimensionless Poisson, continuity and motion equations

$$\begin{aligned} &\frac{\partial v_{1,2}}{\partial t} = -v_{1,2}\frac{\partial v_{1,2}}{\partial x} - \frac{\partial \varphi_{1,2}}{\partial x}, \\ &\frac{\partial \rho_{1,2}}{\partial t} = -\frac{\partial(\rho_{1,2} v_{1,2})}{\partial x}, \\ &\frac{\partial^2 \varphi_{1,2}}{\partial x^2} = -\alpha_{1,2}^2(\rho_{1,2}-1), \end{aligned} \quad (6)$$

with the boundary conditions

$$v_{1,2}(0,t) = 1, \quad \rho_{1,2}(0,t) = 1, \quad \varphi_{1,2}(0,t) = 0, \quad (7)$$

where φ is the dimensionless potential of the electric field, ρ and v are the dimensionless density and velocity of the electron beam $0 \le x \le 1$, the indices 1 и 2 correspond to the drive and response coupled beam-plasma systems, respectively [17], [18]. The unidirectional coupling between such systems is realized by the modification of the boundary conditions on the right boundary of the systems, in the same way as it has been done in [19]

$$\begin{cases} \varphi_1(1,t) = 0 \\ \varphi_2(1,t) = \varepsilon(\rho_2(x=1,t) - \rho_1(x=1,t)) + D\xi(t), \end{cases} \quad (8)$$

The term $D\xi(t)$ corresponds to the noise influence on the system, where $\xi(t)$ is stochastic Gaussian process with zero mean and unit variance, D is the noise intensity. Continuity and motion equations of (6) have been integrated numerically with the help of the one-step explicit two-level scheme with upstream differences and the Poisson equation has been solved by the method of the error vector propagation [20]. The time and space integration steps have been taken as $\Delta t = 0.003$ and Δx=0.005, respectively. The control parameters of Pierce diodes have been chosen as $\alpha_1 = 2.858\pi$ and $\alpha_2 = 2.860\pi$. As in the case of Rössler systems described above the phase synchronization has been detected by the verification of phase locking condition (2). The phases of the drive and response Pierce diodes have been introduced into consideration as rotation angles on $\rho_{1,2}(x = 0.2, t)$, $\rho_{1,2}(x = 0.6, t)$)-planes as well as it has been done in [21], [22].

The numerical simulation of system (6) with the boundary conditions (7)-(8) shows that as in the case of system (1) the synchronous regime in two unidirectional coupled Pierce diodes starts destructing quickly with the growth of the noise intensity. At the same time, due to the specificity of the system itself, the Pierce diodes are more sensible to the influence of noise in comparison with the Rössler systems [23]. Therefore, the new effects for such spatially extended media are revealed for a relatively small value of the noise intensity.

Fig. 4 illustrates the behavior of the response Pierce diode on the plane (x', y') rotating with the frequency of the drive Pierce diode defined by (3) with $x_2 = \rho_2(x = 0.2, t)$, $y_2 = \rho_2(x = 0.6, t)$ in different regimes. It is clearly seen that Fig. 4 is qualitatively identical to Fig. 1. In particular, if the noise intensity is small enough in the synchronous regime the response system attractor on the rotating plane looks like a smeared fixed point which does not envelop the origin (see Fig. 4 (a) and compare it with Fig. 1 (a)). The reconstructed attractor in this case is phase-coherent. Near the boundary of the phase synchronization regime in the case of the small values of the noise intensity the eyelet intermittency takes place. In this case the response system attractor is also phase-coherent, and the phase trajectory on the rotating plane represents the smeared limit cycle which does not touch the origin (Fig. 4 (b)).

The behavior of the Pierce diodes is changed dramatically if the noise intensity becomes a big enough (Figs. 4 (c) and d)). Both for the coupling parameter values corresponding to the synchronous regime in the absence of noise as well as for the asynchronous one the response system attractor is phase-incoherent (Figs. 4 (c) and (d)). At that, if in the absence of noise in the system under study the synchronous regime is realized in the same system in the presence of noise of relatively large amplitude the ring intermittency takes place. The response system attractor on the rotating plane looks like a smeared fixed point enveloping origin in this case (Fig. 4 (c)). For the coupling parameter value corresponding to the eyelet intermittency in the absence of noise in the Pierce diodes subjected to the strong noise influence the coexistence of eyelet and ring intermittencies takes place. The response system attractor in such regime is represented by a smeared limit cycle enveloping origin (Fig. 4 (d)).

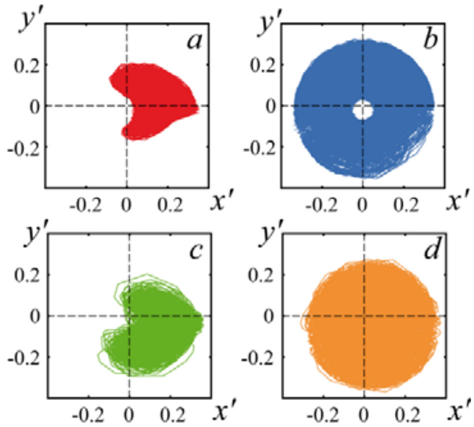

Fig. 4 Phase trajectories of the response Pierce diode on the rotating plane (x', y'): (*a*) $\varepsilon = 0.058$, $D = 10^{-5}$ - the phase synchronization regime, (*b*) $\varepsilon = 0.006$, $D = 10^{-5}$ - eyelet intermittency, (*c*) $\varepsilon = 0.058$, $D = 0.03$ - ring intermittency, (*d*) $\varepsilon = 0.006$, $D = 0.03$ - intermittency of eyelet and ring intermittencies

IV. Conclusion

So, in two unidirectional coupled dynamical systems noise can induce new effects near the boundary of the phase synchronization regime onset. In particular, if the noise intensity is a great enough the boundary value of the synchronous regime onset starts increasing due to the loss of the phase coherence of the response system attractor. At that, near the boundary of phase synchronization in subcritical region the intermittency of eyelet and ring intermittencies takes place. Such phenomenon possesses a high degree of generality. One can expect that such regularities may be observed in a variety of real systems.

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