PROCEEDINGS OF SPIE

SPIEDigitalLibrary.org/conference-proceedings-of-spie

Numerical simulation of coherent resonance in a model network of Rulkov neurons

Andrey V. Andreev, Anastasia E. Runnova, Alexander N. Pisarchik

Andrey V. Andreev, Anastasia E. Runnova, Alexander N. Pisarchik, "Numerical simulation of coherent resonance in a model network of Rulkov neurons," Proc. SPIE 10717, Saratov Fall Meeting 2017: Laser Physics and Photonics XVIII; and Computational Biophysics and Analysis of Biomedical Data IV, 107172E (26 April 2018); doi: 10.1117/12.2315092

Event: Saratov Fall Meeting 2017, 2017, Saratov, Russian Federation

Numerical simulation of coherent resonance in a model network of Rulkov neurons

Andrey V. Andreev[a], Anastasia E. Runnova[a], Alexander N. Pisarchik[ba]

[a]REC "Artificial Intelligence Systems and Neurotechnology", Yuri Gagarin State Technical University of Saratov, Politechnicheskaya Str. 77, Saratov, 410056, Russia;
[b]Center for Biomedical Technology, Technical University of Madrid, Campus Montegancedo, Pozuelo de Alarcon, Madrid, 28223, Spain

ABSTRACT

In this paper we study the spiking behaviour of a neuronal network consisting of Rulkov elements. We find that the regularity of this behaviour maximizes at a certain level of environment noise. This effect referred to as coherence resonance is demonstrated in a random complex network of Rulkov neurons. An external stimulus added to some of neurons excites them, and then activates other neurons in the network. The network coherence is also maximized at the certain stimulus amplitude.

Keywords: Complex network, Rulkov map, Neural network, coherence

1. INTRODUCTION

As all real systems, the neural systems are noisy. Noise can lead to increase or decrease of order in the dynamical systems under noise.[1–3] To be mentioned here are the effects of noise induced order in chaotic dynamics,[4–6] synchronization by external noise,[7,8] and stochastic resonance.[9–12] Also, noise has been shown to play a stabilizing role in ensembles of coupled oscillators and maps.[13,14] Especially interesting is the phenomenon of stochastic resonance, which appears when a nonlinear system is simultaneously driven by noise and a periodic signal.[15–18] At a certain noise amplitude the periodic response is maximal.

The interest in mathematical modeling of neuronal synchronization has significantly increased after neurobiological experiments with two electrically coupled neurons,[19] where various synchronous states have been identified. In order to simulate cooperative neuron dynamics, numerous models based on either iterative maps of differential equations in various coupling configurations have been developed.[19] Depending on the coupling strength and synaptic delay time, coupled neurons generate spike sequences that are matching in their timings, or bursts either with lag or anticipation.[20] When three or more oscillators are accounted for a large number of coupling configurations can be realized. In the theory of graphs or complex networks, these basic configurations are called network motifs.

We explore a simple neural model, the Rulkov map.[21,22] Although this model is not explicitly inspired by physiological processes in the membrane, it is capable of generating extraordinary complexity and quite specific neural dynamics (silence, periodic spiking, and chaotic bursting), thus replicating to a great extent most of the experimentally observed regimes,[19] including spike adaptation, routes from silence to bursting mediated by subthreshold oscillations, emergent bursting, phase and antiphase synchronization with chaos regularization,[21] and complete and burst synchronization.

Further author information: (Send correspondence to A.V. Andreev)
A.V. Andreev: E-mail: andreevandrei1993@gmail.com, Telephone: +7 8452 99 88 32

Saratov Fall Meeting 2017: Laser Physics and Photonics XVIII; and Computational Biophysics and Analysis of Biomedical Data IV, edited by Vladimir L. Derbov, Dmitry E. Postnov, Proc. of SPIE Vol. 10717, 107172E · © 2018 SPIE · CCC code: 1605-7422/18/$18 · doi: 10.1117/12.2315092

2. THE MODEL

Each neuron-like Rulkov element is described by the following system of equations with synaptic coupling:[22]

$$x_{n+1} = f(x_n, x_{n-1}, y_n + \beta_n), \tag{1}$$

$$y_{n+1} = y_n - \mu(x_n + 1) + \mu\sigma + \mu\sigma_n + \mu A^{\xi}\xi_n, \tag{2}$$

where x is a fast variable associated with membrane potential, y is a slow variable which has some analogy with gating variables, the parameters α, σ and $0 < \mu \leq 1$ control individual dynamics of the system, ξ is a Gaussian noise with a zero mean and standard deviation that equals 1, A^{ξ} is noise amplitude. β_n and σ_n are related to external stimuli, f is a piecewise function defined as

$$f(x_n, x_{n-1}, y_n) = \begin{cases} \alpha/(1 - x_n) + y_n, & \text{if } x_n \leq 0 \\ \alpha + y_n, & \text{if } 0 < x_n < \alpha + y_n \text{ and} \\ & x_{n-1} \leq 0 \\ -1, & \text{if } x_n \geq \alpha + y_n \text{ or} \\ & x_{n-1} > 0 \end{cases} \tag{3}$$

It is constructed in a way to reproduce different regimes of neuron-like activity, such as spiking, bursting and silent regimes.

The parameters β_n and σ_n are defined as

$$\beta_n = \beta^e I_n^{ext} + \beta^{syn} I_n^{syn}, \tag{4}$$

$$\sigma_n = \sigma^e I_n^{ext} + \sigma^{syn} I_n^{syn}. \tag{5}$$

Coefficients β^e and σ^e are used to balance the effect of external current I_n^{ext}. β^{syn} and σ^{syn} are coefficients of synaptic coupling. I_n^{syn} is a synaptic current:

$$I_{n+1}^{syn} = \gamma I_n^{syn} - g_{syn} * \begin{cases} (x_n^{post} - x_{rp}), & \text{spike}^{pre}, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

where g_{syn} is the strength of synaptic coupling, $g_{syn} \geq 0$. Indexes *pre* and *post* correspond presynaptic and postsynaptic variables respectively. The first condition in (6) corresponds to the presynaptic impulse (spike) generation time moments and defined as $x_n^{pre} \geq \alpha + y_n^{pre} + \beta_n^{pre}$. Parameter γ is a relaxation time of the synapse, $0 \leq \gamma \leq 1$. It defines the part of synaptic current which preserve as in the next iteration. x_{rp} is a reversal potential that determines the type of the synapse: inhibitory or excitatory.

In our modeling we take values of the parameters $\alpha = 3.65$, $\sigma = 0.06$ and $\mu = 0.0005$ so that each neuron being autonomous demonstrates silent regime dynamics. Also we assume $\beta^e = 0.133$, $\sigma^e = 1.0$, $\beta^{syn} = 0.1$, $\sigma^{syn} = 0.5$ and $x_{rp} = 0.0$. Investigation system is a motif of N neurons coupled to each other with a random coupling strength g_{syn} and relaxation time γ. The values of them are randomly chosen from 0.0 to 0.1 and from 0.0 to 0.5 respectively. In the investigating system we apply an external stimulus to Na neurons. Stimulus is a current impulse of the following form: from the start it equals to 0, at the moment t_s when we apply it current starts equal to A. The values of variables are chosen so that without the external stimulus each neuron is in a silent regime but with starting the application of stimulus excited neurons start periodically generate spikes.

3. THE RESULTS

From the system we take signals as time series of fast variable x from all neurons. Additionally we calculate signal averaging over all neurons and analyse them. In figure 1 one can see these signals for systems of 100 neurons for different values of external stimulus amplitude. On them one can see phenomenon of grouping. It consists in periodically spiking unexcited neurons so that one can see areas of time on time series (d, e, f) where all unstimulated neurons spike and areas where they all are silent and these areas periodically follows one by one.

Increasing the stimulus amplitude leads to increasing frequency of grouping and grouping durations and decreasing time range between them. Also one can see decreasing oscillation amplitude of average signal.

In figure 2 one can see dependencies of time series of x from internal moise amplitude. Increasing noise amplitude leads to decline of grouping effect. Also one can see oscillations in time area where external stimulus amplitude $A = 0$ so noise starts excite neurons without any other stimulus.

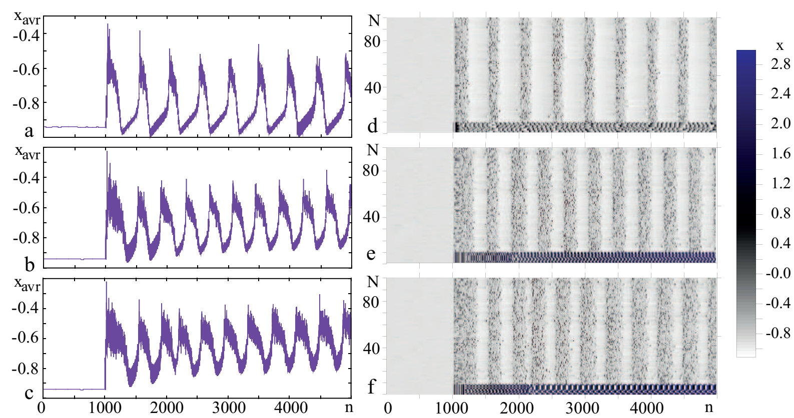

Figure 1. Time series of x variable averaging over all neurons (a), (b), (c) and time series of x variable for all 100 neurons where amplitude x is defined by color (d), (e), (f) for values of external stimulus amplitude $A = 0.5, 1.5$ and 2.5 respectively, $A^{\xi} = 0.1$. We apply the external effect at the first 10 neurons, $N = 100$.

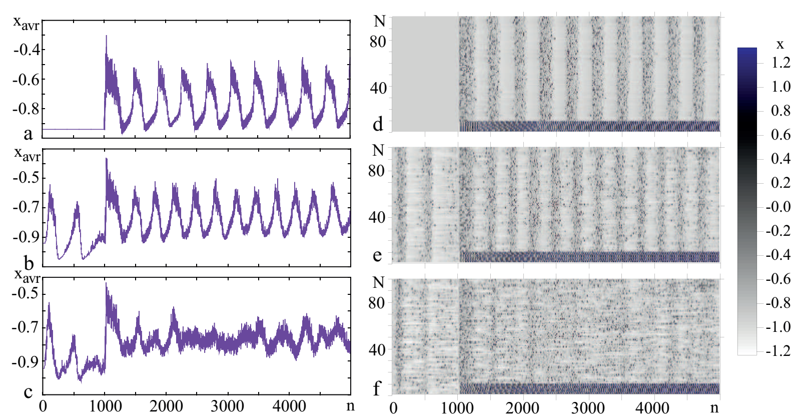

Figure 2. Time series of x variable averaging over all neurons (a), (b), (c) and time series of x variable for all 100 neurons where amplitude x is defined by color (d), (e), (f) for values of noise amplitude $A^{\xi} = 0.0, 1.0$ and 2.0 respectively, $A = 1.0$. We apply the external effect at the first 10 neurons, $N = 100$.

For analyse phenomenon of periodical grouping we calculate dependencies of signal-to-noise ratio (SNR) from amplitude of external stimulus A and amplitude of internal noise A^{ξ}. SNR measured from power spectra of average signal in dB as an excess of main frequency amplitude over background noise.[23,24]

Figure 3,a shows signal-to-noise ratio dependence from external stimulus amplitude, on which one can see the phenomenon of coherent resonance when for a certain values of external stimulus amplitude ($A = 1.3 - 1.6$)

SNR takes the maximum value. For $A > 1.6$ signal-to-noise ratio doesn't change. Decreasing external stimulus amplitude from 1.3 to 0 leads to decreasing SNR. From the power spectra (Fig. 3, b-e) one can see that the main frequency stays the same for all values of external amplitude.

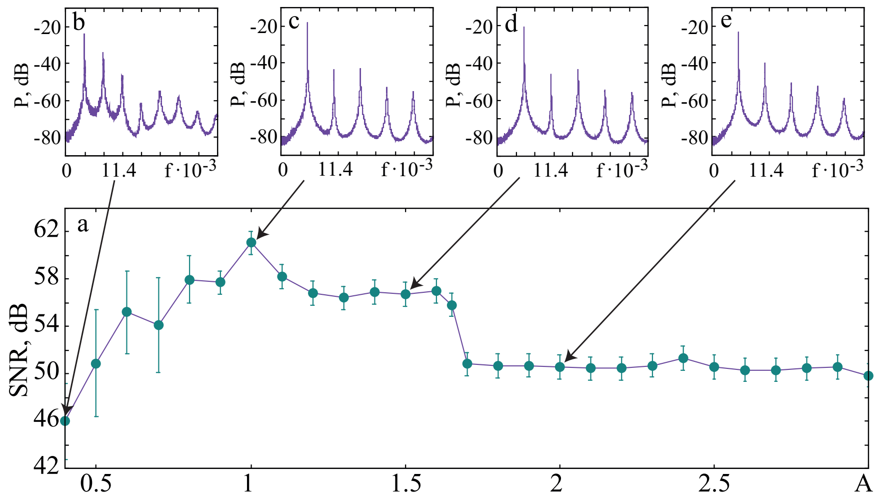

Figure 3. (a) Signal-noise ratio (SNR) versus stimulus amplitude A for $A^{\xi} = 0.1$, $Na = 10$, and $N = 100$ and (b-e) average power spectra for (b) $A = 0.4$, (c) $A = 1$, (d) $A = 1.5$, and (e) $A = 2$.

A distinct behavior occurs in the dependence of SNR on the noise amplitude. As seen from Fig. 4, the SNR has a maximum value in the noiseless network and decreases when the noise amplitude is increased. This means that the network coherence is better without noise.

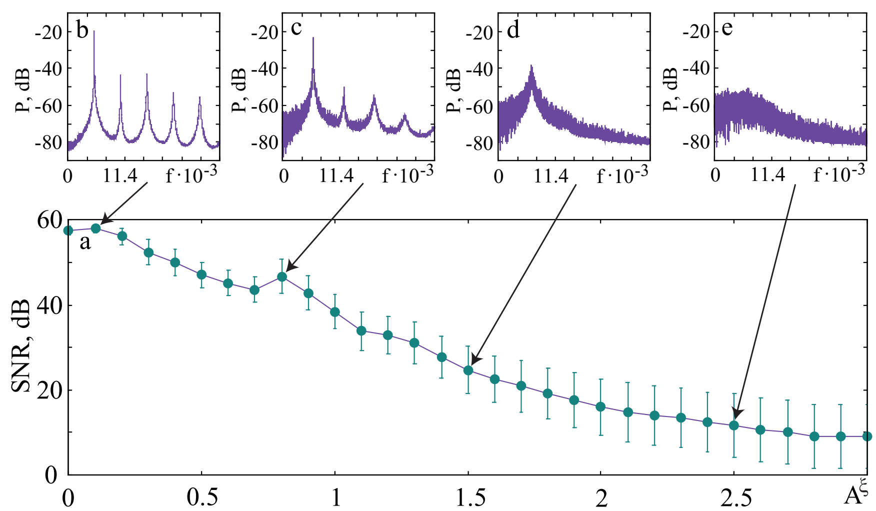

Figure 4. (a) Signal-noise ratio (SNR) versus noise amplitude A^{ξ} for $A = 1$, $Na = 10$, and $N = 100$ and (b-e) average power spectra for (b) $A^{\xi} = 0.1$, (c) $A^{\xi} = 0.8$, (d) $A^{\xi} = 1.5$, and (e) $A^{\xi} = 2.5$.

Following the approach proposed by Pikovsky and Kurths,[15] we characterized the coherence by the correlation time defined as

$$\tau_c = \sum_{n_0}^{T} C(\tau)^2, \tag{7}$$

where n_0 is the number of iterations corresponding to transients, T is the total number of iterations in time series, $C(\tau)$ is the autocorrelation function given as

$$C(\tau) = \frac{\left\langle \left(x_{avr}(n) - \langle x_{avr}\rangle\right)\left(x_{avr}(n+\tau) - \langle x_{arv}\rangle\right) \right\rangle}{\left\langle \left(x_{avr}(n) - \langle x_{avr}\rangle\right)^2 \right\rangle}, \tag{8}$$

where $\langle ... \rangle$ is the time average after transients. The larger the correlation time, the better the coherence.

In Fig. 5 we plot the dependence of the correlation time on the stimulus amplitude A and noise amplitude A^{ξ}. The resonance behavior is clearly seen in all graphics. Comparing results from Figs. 3, 4 and 5 one can see that characteristic correlation time behaves the same way as signal-to-noise ratio. According to the[15] this confirms the presence of coherent resonance phenomenon in the system.

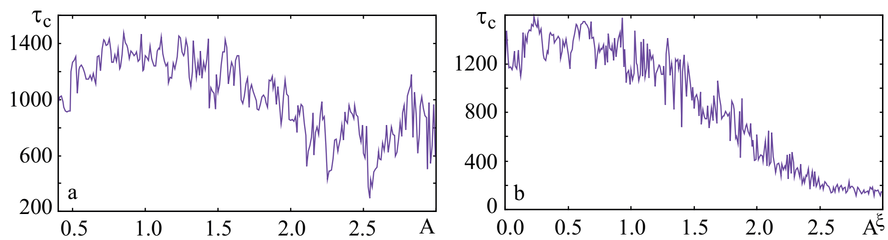

Figure 5. Characteristic correlation time τ_c versus (a) stimulus amplitude A ($A^{\xi} = 0.1$, $N = 100$, $Na = 10$) and (b) noise amplitudes ($A = 1$, $N = 100$, $Na = 10$).

4. CONCLUSION

The macroscopic signal from motif of Rulkov elements with random coupling between them and internal noise presence under external stimulus demonstrates phenomenon of grouping when all unexcited neurons start spiking periodically during the time interval. And at the averaging signal from all neurons we see periodically grouping. Changing such parameters as amplitudes of external stimulus and internal noise we can see phenomenon of coherent resonance when at the certain values of these parameters signal-to-noise ratio takes the maximal values.

ACKNOWLEDGMENTS

This work has been supported by the Russian Science Foundation (Grant No. 17-72-30003).

REFERENCES

[1] Hramov, A. E., Koronovskii, A. A., Kurovskaya, M. K., Ovchinnikov, A. A., and Boccaletti, S., "Length distribution of laminar phases for type-I intermittency in the presence of noise," *Phys. Rev. E* **76**(2), 026206 (2007).

[2] Hramov, A. E., Koronovskii, A. A., and Kurovskaya, M. K., "Zero lyapunov exponent in the vicinity of the saddle-node bifurcation point in the presence of noise," *Phys. Rev. E* **78**, 036212 (2008).

[3] Hramov, A. E., Khramova, A. E., Koronovskii, A. A., and Boccaletti, S., "Synchronization in networks of slightly nonidentical elements," *IJBC* **18**(3), 258–264 (2008).

[4] Matsumoto, K. and Tsuda, I., "Noise-induced order," *J. Stat. Phys.* **31**, 87–106 (1983).

[5] Moskalenko, O. I., Hramov, A. E., Koronovskii, A. A., and Ovchinnikov, A. A., "Effect of noise on generalized synchronization of chaos: theory and experiment," *Europhysics Journal B* **82**(1), 69–82 (2011).

[6] Moskalenko, O. I., Koronovskii, A. A., and Hramov, A. E., "Generalized synchronization of chaos for secure communication: Remarkable stability to noise," *Phys. Lett. A* **374**, 2925–2931 (2010).

[7] Hramov, A. E., Koronovskii, A. A., and Popov, P. V., "Incomplete noise-induced synchronization of spatially extended systems," *Phys. Rev. E* **77**(3), 036215 (2008).

[8] Moskalenko, O. I., Koloskova, A. D., Zhuravlev, M. O., Koronovskii, A. A., and Hramov, A. E., "Intermittent phase synchronization in human epileptic brain," *Proc. SPIE* **10063**, 1006316–1006316 (2017).

[9] Andreev, A. V., Makarov, V. V., Runnova, A. E., Pisarchik, A. N., and Hramov, A. E., "Coherence resonance in stimulated neuronal network," *Chaos Solitons & Fractals* **106**, 80–85 (2018).

[10] Benzi, R., Sutera, A., and Vulpiani, A., "The mechanism of stochastic resonance," *J. Phys. A* **14**, 453–457 (1981).

[11] Jung, P., "Periodically driven stochastic systems," *Phys. Rep.* **234**, 175–295 (1993).

[12] Moss, F., Pierson, D., and O'Gorman, D., "Stochastic resonance: Tutorial and update," *Int. J. Bifurcation Chaos* **4**, 1383–1397 (1994).

[13] Prokhorov, M. D., Ponomarenko, V. I., Kulminskiy, D. D., Koronovskii, A. A., Moskalenko, O. I., and Hramov, A. E., "Resistant to noise chaotic communication scheme exploiting the regime of generalized synchronization," *Nonlinear Dynamics* **87**(3), 2039–2050 (2017).

[14] Hakim, V. and Rappel, W.-J., "Noise-induced periodic behaviour in the globally coupled complex ginzburg-landau equation," *Europhys. Lett.* **27**, 637 (1994).

[15] Pikovsky, A. and Kurths, J., "Coherence resonance in a noise-driven excitable system," *Phys. Rev. Lett.* **78**, 775–779 (1997).

[16] Neiman, A., Saparin, P., and Stone, L., "System size coherence resonance in coupled fitzhugh-nagumo models," *Europhys. Lett.* **61**, 162–167 (2003).

[17] Neiman, A., Saparin, P., and Stone, L., "Coherence resonance at noisy precursors of bifurcations in nonlinear dynamical systems," *Phys. Rev. E* **56**, 270–273 (1997).

[18] Hramov, A. E., Koronovskii, A. A., and Moskalenko, O. I., "Are generalized synchronization and noise-induced synchronization identical types of synchronous behavior of chaotic oscillators?," *Phys. Lett. A* **354**(5–6), 423–427 (2006).

[19] Elson, R. C., Selverston, A. I., Huerta, R., Rulkov, N. F., Rabinovich, M. I., and Abarbanel, H. D. I., "Synchronous behavior of two coupled biological neurons," *Phys. Rev. Lett.* **81**, 5692–5695 (1998).

[20] Lang, X., Lu, Q., and Kurths, J., "Phase synchronization in noise-driven bursting neurons," *Phys. Rev. E* **82**, 021909 (2010).

[21] Rulkov, N. F., "Modeling of spiking-bursting neural behavior using two-dimensional map," *Phys. Rev. E* **65**, 041922 (2002).

[22] Rulkov, N. F., Timofeev, I., and Bazhenov, M., "Oscillations in large-scale cortical networks: Map-based model," *J. Comp. Neuroscience* **17**, 203–223 (2004).

[23] Campos-Meja, A., Pisarchik, A. N., and Arroyo-Almanza, D. A., "Noise-induced on-off intermittency in mutually coupled semiconductor lasers," *Chaos, Solitons and Fractals* **53**, 96–100 (2013).

[24] Hramov, A. E., Koronovskii, A. A., Moskalenko, O. I., Zhuravlev, M. O., Jaimes-Reategui, R., and Pisarchik, A. N., "Separation of coexisting dynamical regimes in multistate intermittency based on wavelet spectrum energies in an erbium-doped fiber laser," *Phys. Rev. E* **93**, 052218 (2016).